Model Question Paper with effect from 2017-18

USN | | | | | | | | | | | **15MAT661**

Sixth Semester B.E.(CBCS) Examination
Linear Algebra
(Open Elective)

Time: 3 Hrs Max.Marks: 80

Note: Answer any FIVE full questions, choosing at least ONE question from each module.

Module-I

1. (a) Find the rank of the matrix by using elementary row operations ; $\begin{bmatrix} 4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \\ 2 & 3 & 1 & 4 \end{bmatrix}$ **(05 Marks)**

   (b) Test for consistency and solve the system of linear equations **(05 Marks)**
   $x + 2y + z = 3$, $2x + 3y + 3z = 10$, $3x - y + 2z = 13$

   (c) Solve the following system of linear equations by LU decomposition method
   $3x + 4y + 5z = 18,$ $2x - y + 8z = 13$, $5x - 2y + 7z = 20$ **(06 Marks)**

OR

2. (a) Reduce the following matrix to row reduced echelon form $\begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$ **(05 Marks)**

   (b) Find the inverse of the matrix using elementary row operations A= $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 3 & 2 \end{bmatrix}$ **(05 Marks)**

   (c) Investigate the values of λ and μ such that the system of equations ; x +y +z = 6 , x +2y +3z = 10 , x +2y + λ z = μ may have (i) Unique solution, (ii)Infinite solution, (iii)No solution.
   (06 Marks)

Module-II

(05 Marks)

3. (a) Define a vector space and give one example.

   (b) Find the co-ordinate vector of (10 ,5, 0) relative to the vectors (1, -1 ,1) , (0 ,1, 2) and (3 ,0, -1).
   (05 Marks)

   (c) If $\alpha_1, \alpha_2, \alpha_3$ are linearly independent in$V_n(R)$, then prove that the vectors $\alpha_1 + \alpha_2$, $\alpha_2 + \alpha_3$, $\alpha_1 + \alpha_3$ are also linearly independent in $V_n(R)$. **(06 Marks)**

OR

4. (a) Prove that the set W = {(x, y, z) / x - 3y + 4z = 0}of the vector space$V_3(R)$ is a subspace of $V_3(R)$. **(05 Marks)**

(b) Find a basis for the subspace of R^4 spanned by the vectors
{(1, 0,−2, 1), (2,−1, 2, 1), (1, 1, 1, 1), (0, 1, 0, 1), (0, 1, 1, 0)}. **(05 Marks)**

(c) Define a subspace. Prove that the intersection of two subspaces of a vector space V(F) is a subspace of V(F). **(06 Marks)**

Module-III

5. (a) Let T: $R^3 \to R^3$ be a linear operator defined by T(a, b, c) = (3a , a – b , 2a +b + c). **(05 Marks)**
Prove that (T^2 – I)(T – 3I) = 0 .

(b) Let T: $R^2 \to R^2$ be a linear transformation such that T(2,3) = (1,0) and T(3,2) = (1,-1). **(05 Marks)**
Find the matrix representation of T.

(c) Let T: $R^3 \to R^3$ be a linear transformation defined by T(x, y, z) = (x+2y - z , y + z, x + y-2z).
Find the basis and dimension of (i) image of T, (ii) kernel of T. **(06 Marks)**

OR

6. (a) If T: V → W be a linear transformation, prove that R(T) is a subspace of W. **(05 Marks)**

(b) Let T: V → W be a linear transformation defined by T(x, y, z) = (x + y , x - y, 2x + z). Find the rank and nullity of T. **(05 Marks)**

(c) Find the matrix of the linear transformation $T:V_2(R) \to V_3(R)$ defined by T(x, y) = (x + y , x, 3x - y) with respect to bases B_1 = {(1,1), (3,1)} and B_2 = { (1,1,1) ,(1,1,1) ,(1,0,0) }. **(06 Marks)**

Module-IV

7. (a) Define an inner product space. For any vectors α , β in an inner product space V ,

prove that $\| \alpha + \beta \| \leq \|\alpha\| + \|\beta\|$. **(05 Marks)**

(b) Prove that an orthogonal set of non zero vectors is linearly independent. **(05 Marks)**

(c) Apply the Gram-Schmidt orthogonalization process to find an orthonormal basis for the subspace of R^4 spanned by the vectors v_1 = (1,1,1,1) , v_2 = (1,2,4,5) , v_3 = (1,-3,-4,-2). **(06 Marks)**

OR

8 (a) If V is an inner product space , then for any vectors α , β in V and any scalar C , prove that **(05 Marks)**

(i) $\|C\alpha\| = |C|\,\|\alpha\|$ (ii) $\|(\alpha,\beta)\| \le \|\alpha\|\,\|\beta\|$

(b) Prove that every finite dimensional inner product space has an orthonormal basis. **(05Marks)**

(c) Find the QR-decomposition of the matrix

$$A=\begin{bmatrix}1 & 0 & 0\\ 1 & 1 & 0\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$ **(06 Marks)**

Module-V

9. (a) Diagonalize the matrix A, given that $A=\begin{bmatrix}-19 & 7\\ -42 & 16\end{bmatrix}$ **(05marks)**

(b) Find the minimum and maximum values of $Q(x) = 9x^2 + 4y^2 + 3z^2$ subject to the constraint $x^T x = 1$. **(05 marks)**

(c) Find the singular value decomposition of $A = \begin{pmatrix}4 & 11 & 14\\ 8 & 7 & -2\end{pmatrix}$ **(06 marks)**

OR

10.(a) Make a change of variable x = p y that transforms the quadratic form $x_1^2 - 8x_1^2 x_2^2 - 5x_2^2$, into a quadratic form with no cross product term . **(05 marks)**

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(b) Orthogonally diagonalize the matrix $A = \begin{pmatrix}3 & -2 & 4\\ -2 & 6 & 2\\ 4 & 2 & 3\end{pmatrix}$ **(05 marks)**

(c) Find the singular value decomposition of $A = \begin{pmatrix}1 & -1\\ -2 & 2\\ 2 & -2\end{pmatrix}$ **(06 marks)**

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Linear Algebra (Open Elective)		
Subject Code: 15MAT661		**IA Marks:** 20
Number of Lecture Hours/Week: 03		**Exam Marks:** 80
Total Number of Lecture Hours: 40		**Exam Hours:** 03
	CREDITS – 03	

Course objectives:

This course will enable students to:

- Represent a system in the form of Linear equations.
- Find the solution of the system of Linear equations using matrix operations.
- Identify Vector spaces and subspaces and their properties.
- Transform a Vector space of one dimension to higher/another dimension.
- Decompose a given matrix using different techniques.

Modules	Teaching Hours	Revised Bloom's Taxonomy (RBT) Level
Module -1		
Linear Equations: System of linear equations, and its solution sets; elementary row operations and echelon forms; matrix operations; invertible matrices, LU-decomposition. (Text.2 Chap.1)	08 Hours	L2, L3, L4
Module -2		
Vector Spaces: Vector spaces; subspaces; bases and dimension; coordinates; summary of row-equivalence; computations concerning subspaces. (Text.1 Chap. 2)	08 Hours	L2, L3, L4
Module -3		
Linear Transformations: Linear transformations; algebra of linear transformations-rank & nullity; representation of transformations by matrices; linear functionals; inverse of a linear transformation. (Text.2 Chap.3)	08 Hours	L1, L2, L3
Module -4		
Inner Product Spaces: Inner products; inner product spaces; orthogonal sets and projections; Gram-Schmidt process; QR-decomposition. (Ref.1 Chap. 8)	08 Hours	L2, L3, L4
Module -5		
Symmetric Matrices and Quadratic Forms: Diagonalization; quadratic forms; constrained optimization; singular value decomposition. (Text.2 Chap.7)	08 Hours	L1,L2, L3, L4

RBT Levels:- L1 = Remembering , L2 = Understanding, L3 = Applying, L4 = Analyze		

Course Outcomes:

At the end of the course Student will be able to,

- Analyze whether a system is consistent or inconsistent.
- Find whether the solution of the system is unique or infinite.
- Perform row operations on matrices and find bases and dimension.
- Linearly transform the system from one dimension to another and find the definition of Linear transformation.
- Compute orthogonal and orthonormal vectors.
- Find the solution to an inconsistent system using Least Square solutions.

Graduating Attributes (as per NBA):

- Engineering Knowledge
- Problem Analysis
- Design / development of solutions(partly)
- Investigations

Question paper pattern:

- The question paper will have ten questions.
- Each full Question consisting of 16 marks. There will be 2 full questions (with a maximum of four sub questions) from each module.
- Each full question will have sub questions covering all the topics under a module.
- The students will have to answer 5 full questions, selecting one full question from each module.

TEXT BOOKS:

1. David C. Lay, “Linear Algebra and its Applications,” 3rd edition, Pearson Education (Asia) Pte. Ltd,2005.
2. Kenneth Hoffman and Ray Kunze, "Linear Algebra," 2nd edition, Pearson Education (Asia) Pte. Ltd/2004.

REFERENCE BOOKS:

1. Bernard Kolman and David R. Hill, "Introductory Linear Algebra with Applications”, Pearson Education (Asia) Pte. Ltd, 7th edition, 2003.
2. Gilbert Strang, "Linear Algebra and its Applications”, 3rd edition, Thomson Learning Asia, 2003.