Model Question Paper-II with effect from 2016-17

USN | | | | | | | | | | | **15MAT41**

Fourth Semester B.E.(CBCS) Examination
Engineering Mathematics-IV

(Common to all Branches)

Time: 3 Hrs   Max.Marks: 80

Note: Answer any FIVE full questions, choosing at least ONE question from each module. Use of statistical tables allowed.

Module-I

1. (a) Solve $\frac{dy}{dx} = x^2y^2 + 1, y(0) = 1$ using Taylor's series method considering up to fourth degree terms and, find the $y(0.1)$ **(05 Marks)**

(b) Use Runge - Kutta method of fourth order to solve $10\frac{dy}{dx} = x^2 + y^2, y(0) = 1$,to find $y(0.2)$. (Take $h = 0.2$) . **(05 Marks)**

(c) Given that $\frac{dy}{dx} = x(1 + y^2)$ and $y(1) = 2, y(2.1) = 1.2330, y(2.2) = 1.5480, \& \ y(2.3) = 1.9790$ find $y(1.4)$, using Adam-Bashforth predictor-corrector method. **(06 Marks)**

OR

2. (a) Solve the differential equation $\frac{dy}{dx} = -xy^2$ under the initial condition $y(0) = 2$ by using modified Euler's method at the point $x = 0.1$. Perform three iterations at each step, taking $h = 0.05$. **(05 Marks)**

(b) Use fourth order Runge - Kutta method, to find $y(0.2)$, given $\frac{dy}{dx} = 3x + y, y(0) = 1.$ **(05 Marks)**

(c) Apply Milne's predictor-corrector formulae to compute $y(1.2)$ given **(06 Marks)**

$\frac{dy}{dx} = 3x - 4y^2$ with

x	0	0.3	0.6	0.9
y	1.0	1.3020	1.3795	1.4762

Module-II

3. (a) By Runge - Kutta method , solve $\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$ for $x = 0.2$, correct to four decimal places, using initial conditions $y(0) = 1, y'(0) = 0.$ **(05 Marks)**

(b) If α and β are two distinct roots of $J_n(x) = 0,$ then prove that $\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx = 0$ if $\alpha \neq \beta.$ **(05 Marks)**

(c) Express $f(x) = x^3 - 5x^2 + 14x + 5$ in terms of Legendre polynomials. **(06 Marks)**

OR

4. (a) Apply Milne's predictor-corrector method to compute $y(0.4)$ given the differential equation $\frac{d^2y}{dx^2}+\frac{dy}{dx}=2e^x$ and the following table of initial values: **(05 Marks)**

x	0	0.1	0.2	0.3
y	2	2.01	2.04	2.09
y'	0	0.20	0.40	0.60

(b) With usual notation, show that $J_{-1/2}(x)=\sqrt{(2/\pi x)}\cos x$. **(05 Marks)**

(c) With usual notation, derive the Rodrigues's formula viz., $P_n(x)=\frac{1}{2^n n!}\frac{d^n}{dx^n}\left(x^2-1\right)^n$. **(06 Marks)**

Module-III

5. (a) Derive Cauchy-Riemann equation in cartesian form. **(05 Marks)**

(b) evaluate $\int_C \frac{\sin\pi z^2+\cos\pi z^2}{(z-1)^2(z-2)}dz$ where C is the circle $|z|=3$, using Cauchy's residue theorem. **(05 Marks)**

(c) Discuss the transformation $w=z^2$. **(06 Marks)**

OR

6. (a) Find the analytic function whose real part is $r^2\cos 2\theta$. **(05 Marks)**

(b) State and prove Cauchy's theorem. **(05 Marks)**

(c) Find the bilinear transformation which maps the points $z=\infty, i, 0$ into the points $w=-1,-i,1$. **(06 Marks)**

Module-IV

7. (a) Derive mean and variance of the Poisson distribution. **(05 Marks)**

(b) A random variable X has the following probability function for various values of x:

$X(=x_i)$	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Find (i) the value of k (ii) $P(x<6)$ (iii) $P(x\geq 6)$ **(05 Marks)**

(c) Let X be the random variable with the following distribution and Y is defined by X^2 :

$X(=x_i)$	-2	-1	1	2
$f(x_i)$	1/4	1/4	1/4	1/4

Determine (i) the distribution of g of Y (ii) joint distribution of X and Y (iii) $E(X)$, $E(Y)$, $E(XY)$. **(06 Marks)**

OR

8. (a) When a coin is tossed 4 times find, using binomial distribution, the probability of getting (i) exactly one head (ii) at most 3 heads (iii) at least 3 heads. **(05 Marks)**

(b) In a normal distribution, 31% of the items are under 45 and 8% of the items are over 64%. Find the mean and standard deviation of the distribution. **(05 Marks)**

(c) A fair coin is tossed thrice. The random variables X and Y are defined as follows :

X=0 or 1 according as head or tail occurs on the first; Y= Number of heads.

Determine (i) the distribution of X and Y (ii) joint distribution of X and Y. **(06 Marks)**

Module-V

9. (a) Define the terms:(i)Null hypothesis (ii)Confidence intervals (iii)Type-I and Type-II errors **(05marks)**

(b) Ten individuals are chosen at random from a population and their heights in inches are found to be 63,63,66,67,68,69,70,70,71,71.Test the hypothesis that the mean height of the universe is 66 inches. ($t_{0.05}$ = 2.262 for 9 d.f.). **(05 marks)**

(c) Show that probability matrix $P = \begin{bmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{bmatrix}$ is regular stochastic matrix and find the associated unique fixed probability vector. **(06 marks)**

OR

9. (a) A manufacture claimed that at least 95%of the equipment which he supplied to a factory conformed to specifications. An examination of a sample of 200 pieces of equipment revealed that 18 of them were faulty. Test his claim at a significance level of 1% and 5%. **(05 marks)**

(b) Explain (i) transient state (ii) absorbing state (iii) recurrent state of a Markov chain. **(05marks)**

(c) Three boys A, B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C. But C is just as likely to throw the ball to B as to A. If C was the first person to throw the ball , find the probabilities that after three throws (i) A has the ball (ii) B has the ball and (iii) C has the ball. **(06 marks)**

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