## Model Question Paper (CBCS) with effect from 2016-17

USN | | | | | | | | | | |

**15MATDIP31**

# Third Semester B.E.(CBCS) Examination
# Additional Mathematics - I
(Common to all Branches)

Time: 3 Hrs — Max.Marks: 80

**Note: Answer any FIVE full questions, choosing at least ONE question from each module**

## Module-I

1. (a) State De Moivre's theorem. Using the same, prove that
$(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n=2^{n+1}\cos^n(\theta/2)$ **(06 Marks)**

(b) Show that $\left(\frac{1+\sin\theta+i\cos\theta}{1+\sin\theta-i\cos\theta}\right)^n=\cos\left(\frac{n\pi}{2}-n\theta\right)+i\sin\left(\frac{n\pi}{2}-n\theta\right)$ **(05 Marks)**

(c) Use De Moivre's theorem to solve the equation $x^7+x^4+x^3+1=0$. **(05 Marks)**

**OR**

2. (a) Define scalar and vector product of two vectors. If $A=2\vec{i}-3\vec{j}-\vec{k}$ and $B=\vec{i}+4\vec{j}-2\vec{k}$,
find $A\cdot B \; and \; A\times B$ **(06 Marks)**

(b) Show that the vectors $A=\vec{i}-2\vec{j}+3\vec{k}$, $B=2\vec{i}+\vec{j}+\vec{k}$ and $C=3\vec{i}+4\vec{j}-\vec{k}$ are coplanar **(05 Marks)**

(c) For any three vectors *A*,*B*,*C* show that $[B\times C, C\times A, A\times B]=[A,B,C]^2$ **(05 Marks)**

## Module-II

3. (a) Find the $n^{th}$ derivative of (i) $\log_{10}\sqrt{(3x+5)^2(2-3x)}$ (ii) $e^{5x}\sin 3x\cos 4x$. **(06 Marks)**

(b)If $y=e^{m\sin^{-1}x}$, prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+m^2)y_n=0.$ **(05 Marks)**

(c) Find the angle between curves : $r=a(1+\cos\theta)$ and $r=b(1-\cos\theta)$. **(05 Marks)**

**OR**

4. (a) If $u=x^2\tan^{-1}(y/x)-y^2\tan^{-1}(x/y)$, show that $[\partial^2u/\partial x\partial y]=[(x^2-y^2)/(x^2+y^2)]$. **(06 Marks)**

(b) If $u=\sin^{-1}[(x^2y^2)/(x+y)]$, prove that $xu_x+yu_y=3\tan u$ ,using Euler's theorem. **(05 Marks)**

(c) If $u=f(x-y,y-z,z-x)$, show that $u_x+u_y+u_z=0$ **(05 Marks)**

## Module-III

5. (a) Obtain a reduction formula for $\int_0^{\pi/2} \sin^n x dx, (n > 0)$. **(06 Marks)**

(b) Evaluate: $\int_0^{\infty} \frac{x^2 dx}{(1+x^2)^3}$ **(05 Marks)**

(c) Evaluate: $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)dydxdz$ **(05 Marks)**

**OR**

6. (a) Obtain a reduction formula for $\int_0^{\pi/2} \cos^n x dx, (n > 0)$ **(06 Marks)**

(b) Evaluate : $\int_0^{2a} x^3\sqrt{2ax - x^2}\, dx$ **(05 Marks)**

(c) Evaluate $\iint_R xydxdy$ where $R$ is the first quadrant of the circle $x^2 + y^2 = a^2, x \geq 0, y \geq 0$. **(05 Marks)**

## Module-IV

7. (a) A particle moves along a curve $x = e^{-t}, y = 2\cos 3t, z = 2\sin 3t$ where $t$ is the time variable. Determine the components of velocity and acceleration vectors at $t = 0$ in the direction of $\vec{i} + \vec{j} + \vec{k}$. **(08 Marks)**

(b) Find the values of the constants $a, b, c$ such that $\vec{F} = (x + y + az)\vec{i} + (bx + 2y - z)\vec{j} + (x + cy + 2z)\vec{k}$ is irrotational. **(08 Marks)**

**OR**

8. (a) If $\vec{F} = (x + y + z)\vec{i} + \vec{j} - (x + y)\vec{k}$, show that $\vec{F} \times curl\vec{F} = 0$ **(06 Marks)**

(b) If $\phi(x, y, z) = x^3 + y^3 + z^3 - 3xyz$, find $\nabla\phi$ & $|\nabla\phi|$ at $(1,-1,2)$ **(05 Marks)**

(c) Find the directional derivative of $\phi(x, y, z) = x^2yz + 4xz^2$ at $(1,-2,1)$ in the direction of $2\vec{i} - \vec{j} - 2\vec{k}$ **(05 Marks)**

## Module-V

9. (a) Solve: $(x^2 + 2xy - y^2)dx + (y^2 + 2xy - x^2)dy = 0$ **(06 Marks)**

(b) Solve: $[y(1 + 1/x) + \cos y]dx + [x + \log x - x\sin y]dy = 0$ **(05 Marks)**

(c) Solve: $(1 + y^2)dx = (\tan^{-1} y - x)dy$ **(05 Marks)**

**OR**

10. (a) Solve: $(x + 2y - 3)dx - (2x + y - 3)dy = 0$ **(06 Marks)**

(b) Solve: $\left[y^2 e^{xy^2} + 4x^3\right]dx + \left[2xye^{xy^2} - 3y^2\right]dy = 0$ **(05 Marks)**

(c) Solve: $(x^3\cos^2 y - x\sin 2y)dx = dy$ **(05 Marks)**

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