Model Question Paper (CBCS) with effect from 2016-17

USN | | | | | | | | | | | **15MAT31**

Third Semester B.E.(CBCS) Examination
Engineering Mathematics-III

(Common to all Branches)

Time: 3 Hrs Max.Marks: 80

Note: Answer any FIVE full questions, choosing at least ONE question from each module

Module-I

1. (a) Find the Fourier series expansion of $f(x)$, if $f(x)=\begin{cases} -\pi, & in \; -\pi \le x < 0 \\ x, & in \; 0 < x \le \pi. \end{cases}$

   Hence deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+...=\frac{\pi^2}{8}$. **(08 Marks)**

   (b) A periodic function $f(x)$ of period '6' is specified by the following table over the interval $(0,6)$:

x	0	1	2	3	4	5	6
$f(x)$	9	18	24	28	26	20	9

   Obtain the Fourier series of $f(x)$ up to second harmonics. **(08 Marks)**

OR

2. (a) Expand the function $f(x)=[(\pi-x)/2]^2$ as a Fourier series in the interval $0 \le x \le 2\pi$.

   Hence deduce that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+...=\frac{\pi^2}{6}$. **(06 Marks)**

   (b) Obtain the Fourier series of $f(x)=|x|$ valid in the interval $(-l,l)$. **(05 Marks)**

   (c) Find the half-range sine series of $f(x)=(x-1)^2$ the interval $0 \le x \le 1$. **(05 Marks)**

Module-II

3. (a) If $f(x)=\begin{cases} 1, & for \; |x| \le 1 \\ 0 & for \; |x| > 1 \end{cases}$, find the infinite Fourier transform of *f(x)* and

   hence evaluate $\int_0^\infty \frac{\sin x}{x}dx$ **(06 Marks)**

   (b) Find the Fourier sine transform of $e^{-|x|}$. Hence show that $\int_0^\infty \frac{x\sin mx}{1+x^2}dx=\frac{\pi e^{-m}}{2}, m.>0.$ **(05 Marks)**

   (c) Find the Z-transform of $(i)\cos n\theta \;\&\; (ii)\sin n\theta$ **(05 Marks)**

OR

4. (a) Using Z-transform, solve $y_{n+2} - 4y_n = 0$, given that $y_0 = 0, y_1 = 2$ **(06 Marks)**

(b) Find the complex Fourier transform of $e^{-a^2x^2}\,(a > 0)$ **(05 Marks)**

(c) Obtain the inverse Z-transform of $18z^2/[(2z-1)(4z+1)]$ **(05 Marks)**

Module-III

5. (a) Calculate the Karl Pearson's coefficient of correlation for 10 students who have obtained the following percentage of marks in Mathematics and Electronics: **(06 Marks)**

Roll No.	1	2	3	4	5	6	7	8	9	10
Marks in Mathematics	78	36	98	25	75	82	90	62	65	39
Marks in Electronics	84	51	91	60	68	62	86	58	53	47

(b) Fit a best fitting parabola $y = ax^2 + bx + c$ for the following data: **(05 Marks)**

x	1	2	3	4	5
y	10	12	13	16	19

(c) Using regula-falsi method compute the real root of the equation $xe^x = 2$, correct to three decimal places. **(05 Marks)**

OR

6. (a) Fit a curve of the form $y = ae^{bx}$ to the following data: **(06 Marks)**

x	77	100	185	239	285
y	2.4	3.4	7.0	11.1	19.6

(b) If θ is the acute angle between the lines of regression, then show that $\tan\theta = \frac{\sigma_x\sigma_y}{\sigma_x^2 + \sigma_y^2}\left(\frac{1-r^2}{r}\right)$.

Explain the significance of $\tan\theta$ when $r = 0$ & $r = \pm 1$. **(05 Marks)**

(c) Find the real root of the equation $x\sin x + \cos x = 0$ near $x = \pi$ correct four decimal places, using Newton- Raphson method. Carryout three iterations **(05 Marks)**

Module-IV

7. (a) From the data given below, find the number of students who obtained (i) less than 45 marks and, (ii) between 40 & 45 marks: **(06 Marks)**

Marks	30-40	40-50	50-60	60-70	70-80
No .of Students	31	42	51	35	31

(b) Using Newton's general interpolation formula, fit an interpolating polynomial for the following data: **(05 Marks)**

x	-4	-1	0	2	5
$f(x)$	1245	33	5	9	1335

(c) Using Simpson's $(1/3)^{rd}$ rule, evaluate $\int_0^1 \frac{dx}{1+x^2}$, taking $h = 1/6$. **(05 Marks)**

OR

8. (a) From the following table, which gives the distance y (in nautical miles) of the visible horizon for the given given heights x (in feet) above the earth' surface, find the value of y at x= 410: **(06 Marks)**

x	100	150	200	250	300	350	400
y	10.63	13.03	15.04	16.81	18.42	19.90	21.27

(b) Use Lagrange's interpolation formula to find $f(4)$,given: **(05 Marks)**

x	0	2	3	6
$f(x)$	-4	2	14	158

(c) Use Weddle's rule to evaluate $\int_0^{\pi/2} \sqrt{\cos\theta}\, d\theta$, dividing $[0, \pi/2]$ into six equal parts. **(05 Marks)**

Module-V

9. (a) Derive Euler's equation in the standard form viz., $\frac{\partial f}{\partial y} - \frac{d}{dx}\left[\frac{\partial f}{\partial y'}\right] = 0$. **(06 Marks)**

(b) Find the curve on which the functional $\int_0^1 \left(y^2 + x^2 y'\right) dx$ with $y(0) = 0, y(1) = 1$,

can be extermized. **(05 Marks)**

(c) If $\vec{F} = 3xy\vec{i} - y^2\vec{j}$, evaluate $\int_C \vec{F} \cdot d\vec{r}$ where C is the arc of parabola $y = 2x^2$

from $(0,0)$ to $(1,2)$. **(05 Marks)**

OR

10. (a) Verify Green's theorem in the plane for $\oint_c \left(xy + y^2\right) dx + x^2 dy$ where C is the closed curve bounded

by $y = x$ & $y = x^2$. **(06 Marks)**

(b) Using Stoke's theorem, evaluate $\iint_s \left(\nabla \times \vec{F}\right) \cdot \hat{n}\, dS$ where $\vec{F} = 3y\vec{i} - xz\vec{j} + yz^2\vec{k}$ and S is the surface of

the paraboloid $2z = x^2 + y^2$ bounded by $z = 2$. **(05 Marks)**

(c) Prove that geodesics on a plane are straight lines. **(05 Marks)**

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