Model Question Paper

ENGINEERING MATHEMATICS - I

(14MAT11)

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing one full question from each module.

MODULE 1

1) a) If $y = e^{m\sin^{-1}x}$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (m^2+n^2)y_n = 0$ (7 marks)

b) Find the angle of intersection between the curves $r^2\sin 2\theta = 4$ and $r^2 = 16\sin 2\theta$ (6 marks)

c) Derive an expression to find radius of curvature in polar form (7 marks)

OR

2) a) If x=sint, y=cosmt, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2-n^2)y_n = 0$ (7 marks)

a) Find the pedal equation , $r^n = a^n \cos n\theta$ (6 marks)

b) Show that the radius of curvature of the curve $x^3 + y^3 = 3axy$ at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ $is - \frac{3a}{8\sqrt{2}}$ (7 marks)

MODULE 2

3) a) Obtain the Maclaurin's series for log(1+sinx) upto the term containing x^4 (7 marks)

b) If u be homogeneous function of degree n in x and y, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$ (6 marks)

c) If u= f(x-y, y-z, z-x) then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ (7 marks)

OR

4) a) Evaluate $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{1/x}$ (6 marks)

a) If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, $show$ that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$ (7 marks)

b) If $u = x+y+z, v = x^2+y^2+z^2, w = xy+yz+zx, then\ find \frac{\partial (u,v,w)}{\partial (x,y,z)}$ (7 marks)

MODULE 3

5) a) A particle moves along the curve $x=1-t^3$, $y=1+t^2$ and z= 2t-5, find the components of velocity and acceleration at t=1 in the direction 2i+j+2k (7 marks)

b) Using differentiation under integral sign, evaluate $\int_0^1 \frac{x^{\alpha}-1}{\log x} dx, \ \alpha \geq 0$ (7marks)

c) State the general rules to trace a polar curve (6 marks)

OR

6) a) Show that $\vec{F} = \frac{xi+yj}{x^2+y^2}$ is both solenoidal and irrotational (7 marks)

a) Show that Curl(grad$\emptyset$)=$\vec{0}$ (6 maeks)

b) State the general rules to trace a cartesian curve (7 marks)

MODUEL 4

7) a) Obtain the reduction formula for $\int_0^{\pi/2} \sin^n x \, dx$ (7 marks)

b) Solve $\left(y^2 e^{xy^2} + 4x^3\right)dx + \left(2xye^{xy^2} - 3y^2\right)dy = 0$ (6 marks)

c) Show that the orthogonal trajectories of a family of circles passing through the origin having centres on x-axis is a family of circles passing through the origin having their cetres on y-axis (7 marks)

OR

8) a) Evaluate $\int_0^{\pi/6} \cos^4 3\theta \sin^3 6\theta \, d\theta$ (7 marks)

b) Solve $x\frac{dy}{dx} + y = x^3 y^6$ (6 marks)

c) If a substance cools from 370k to 330k in 10minutes, when the temperature of the surrounding air is 290k. Find the temperature of the substance after 40 minutes (7 marks)

MODULE 5

9) a) Solve x+4y-z= -5, x+y-6z= -12, 3x-y-z= 4 by Gauss elimination method. (7 marks)

b) Diagonalise the matrix A= $\begin{pmatrix} -19 & 7 \\ -42 & 16 \end{pmatrix}$ (6 marks)

c) Determine the largest eigan value and the corresponding eigan vector of A= $\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$

using Rayleigh's Power method.

(7 marks)

OR

10) a) Solve by LU decomposition method 3x+2y+7z=4, 2x+3y+z=5, 3x+4y+z=7 (7 marks)

b) Show that the transformation $y_1 = 2x_1 - 2x_2 - x_3,\ y_2 = -4x_1 + 5x_2 + 3x_3, y_3 = x_1 - x_2 - x_3$ is regular and find the inverse transformation. (6 marks)

c) Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into canonical form by orthogonal transformation.

(7 marks)