Model Question Paper with effect from 2018-19
(CBCS Scheme)

USN | | | | | | | | | | | **17MAT41**

Fourth Semester B.E.(CBCS) Examination
Engineering Mathematics - IV
(Common to all Branches)

Time: 3 Hrs Max.Marks: 100

Note: Answer any FIVE full questions, choosing at least ONE question from each module. Use of statistical tables allowed.

Module-I

1. (a) Solve $\frac{dy}{dx} = 2y + 3e^x, y(0) = 0$ using Taylor's series method considering up to fourth degree terms and, find the $y(0.1)$ **(06 Marks)**

(b) Use Runge - Kutta method of fourth order to solve $(x+y)\frac{dy}{dx} = 1, y(0.4) = 1$, to find $y(0.5)$. (Take $h = 0.1$). **(07 Marks)**

(c) Given that $\frac{dy}{dx} = x - y^2$ and $y(0) = 1, y(0.1) = 0.9117, y(0.2) = 0.8494, \& \ y(0.3) = 0.8061$ find $y(0.4)$, using Adam-Bashforth predictor-corrector method. **(07 Marks)**

OR

2. (a) Solve the differential equation $\frac{dy}{dx} = x + y^2$ under the initial condition $y(0) = 1$ by using modified Euler's method at the point $x = 0.2$. Perform three iterations at each step, taking $h = 0.1$. **(06 Marks)**

(b) Use fourth order Runge - Kutta method, to find $y(1.2)$, given $\frac{dy}{dx} = xy, y(1) = 2$. **(07 Marks)**

(c) Apply Milne's predictor-corrector formulae to compute $y(0.3)$ given **(07 Marks)**

$\frac{dy}{dx} = x^2 + y^2$ with

x	-0.1	0.0	0.1	0.2
y	0.9087	1.0000	1.1114	1.2525

Module-II

3. (a) Using Runge - Kutta method, solve $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} - 2xy = 1$, for $x = 0.1$, correct to four decimal places, using initial conditions $y(0) = 1, y'(0) = 0$. **(06 Marks)**

(b) If α and β are two distinct roots of $J_n(x) = 0$, then prove that $\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx = 0$ if $\alpha \neq \beta$. **(07 Marks)**

(c) Express $f(x) = x^3 + 2x^2 - x - 3$ in terms of Legendre polynomials. **(07 Marks)**

OR

4. (a) Apply Milne's predictor-corrector method to compute $y(0.4)$ given the differential equation $\frac{d^2y}{dx^2} = 1 + \frac{dy}{dx}$ and the following table of initial values: **(06 Marks)**

x	0	0.1	0.2	0.3
y	1	1.1103	1.2427	1.3990
y'	1	1.2103	1.4427	1.6990

(b) With usual notation, show that (i) $J_{1/2}(x) = \sqrt{(2/\pi x)}\sin x$ (ii) $J_{-1/2}(x) = \sqrt{(2/\pi x)}\cos x$. **(07 Marks)**

(c) With usual notation, derive the Rodrigues's formula viz., $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$. **(07 Marks)**

Module-III

5. (a) State and prove Cauchy's theorem. **(06 Marks)**

(b) Evaluate $\int_C \frac{2z^2+1}{(z+1)^2(z-2)}dz$ where C is the circle $|z| = 3$, using Cauchy's residue theorem. **(07 Marks)**

(c) Discuss the transformation $w = e^z$. **(07 Marks)**

OR

6. (a) Find the analytic function $f(z) = u + iv$, given $v = [r - (1/r)]\sin\theta, r \neq 0$. **(06 Marks)**

(b) Derive Cauchy-Riemann equation in cartesian form. **(07 Marks)**

(c) Find the bilinear transformation which maps the points $z = i, 1, -1$ into the points $w = 1, 0, \infty$. **(07 Marks)**

Module-IV

7. (a) Derive mean and variance of the Binomial distribution. **(06 Marks)**

(b) A random variable X has the following probability function for various values of x:

$X(=x_i)$	-2	-1	0	1	2	3
$P(x)$	0.1	k	0.2	$2k$	0.3	k

Find (i) the value of k (ii) $P(x<1)$ (iii) $P(x \geq -1)$ **(07 Marks)**

(c) A fair coin is tossed thrice. The random variables X and Y are defined as follows :

X=0 or 1 according as head or tail occurs on the first; Y= Number of heads.

Determine (i) the distribution of X and Y (ii) joint distribution of X and Y. **(07 Marks)**

OR

8. (a) Two persons *A* and *B* play a game in which their chances of winning are in the ratio 3:2. If 6 games are played, find *A*'s chance of winning at least three games. **(06 Marks)**

(b) In a normal distribution, 7% of the items are under 35 and 89% of the items are under 63. Find the mean and standard deviation of the distribution. **(07 Marks)**

(c) Let *X* be the random variable with the following distribution and *Y* is defined by X^2

$X(=x_i)$	-2	-1	1	2
$f(x_i)$	$1/4$	$1/4$	$1/4$	$1/4$

Determine (i) the distribution of *g* of *Y* (ii) joint distribution of *X* and *Y* (iii) *E(XY)*. **(07 Marks)**

Module-V

9. (a) A manufacture claimed that at least 95%of the equipment which he supplied to a factory conformed to specifications. An examination of a sample of 200 pieces of equipment revealed that 18 of them were faulty. Test his claim at a significance level of 1% and 5%. **(06 Marks)**

(b) Explain (i) transient state (ii) absorbing state (iii) recurrent state of a Markov chain. **(07 Marks)**

(c) Show that probability matrix $P = \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$ is regular stochastic matrix and find the associated unique fixed probability vector. **(07 Marks)**

OR

10. (a) Define the terms : (i)Null hypothesis (ii)Confidence intervals (iii)Type-I and Type-II errors **(06 Marks)**

(b) The following are the *I.Q.*'s of a randomly chosen sample of 10 boys: 70,120,110,101,88,83,95,98,107,100. Does this supports the hypothesis that the population mean of *I.Q.*'s is 100 at 5% level of significance? ($t_{0.05}$ = 2.262 for 9 d.f.) **(07 Marks)**

(c) Three boys A, B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C. But C is just as likely to throw the ball to B as to A. If C was the first person to throw the ball , find the probabilities that after three throws (i) A has the ball (ii) B has the ball and (iii) C has the ball. **(07 Marks)**

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