USN | | | | | | | | | | |

17MATDIP31

Third Semester B.E.Degree Examination

Additional Mathematics-I

(Common to all Branches)

Time: 3 Hrs Max.Marks: 100

Note: Answer any FIVE full questions, choosing at least ONE question from each module.

Module-I

1. (a) Define dot product between two vectors A and B. Find the sine of the angle between $\vec{A} = 2\vec{i} - 2\vec{j} + \vec{k}$ and $\vec{B} = 2\vec{i} - 2\vec{j} + \vec{k}$. **(08 Marks)**

   (b) Express $\sqrt{5} + 2i$ in the polar form and hence find its modulus and amplitude. **(06 Marks)**

   (c) Find the real part of $\dfrac{1}{1 + \cos\theta + i\sin\theta}$ **(06 Marks)**

OR

2. (a) Show that $(1 + \cos\theta + i\sin\theta)^n + (1 + \cos\theta - i\sin\theta)^n = 2^{n+1}\cos^n(\theta/2)\cos(n\theta/2)$ **(08 Marks)**

   (b) If $\vec{A} = 2\vec{i} - 2\vec{j} + \vec{k}$ and $\vec{B} = 2\vec{i} - 2\vec{j} + \vec{k}$, show that $(\vec{A} + \vec{B})$ and $(\vec{A} - \vec{B})$ are orthogonal. **(06 Marks)**

   (c) Find the value of λ, so that the vectors $\vec{A} = 3\vec{i} + 5\vec{j} - 3\vec{k}$, $\vec{B} = \vec{i} + \lambda\vec{j} + 2\vec{k}$ and $\vec{C} = 2\vec{i} - 2\vec{j} + \vec{k}$ are coplanar. **(06 Marks)**

Module-II

3. (a) If $x = \tan(\log y)$, prove that $(1 + x^2)y_{n+1} + (2nx - 1)y_n + n(n-1)y_{n-1} = 0$. **(08 Marks)**

   (b) Find the angle between the curves $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$. **(06 Marks)**

   (c) Using Euler's theorem, prove that $xu_x + yu_y = \sin 2u$, where $u = \tan^{-1}\left[(x^3 + y^3)/(x + y)\right]$. **(06 Marks)**

OR

4. (a) Obtain the Maclaurin's series expansion of $\log\sec x$ up to the terms containing x^6. **(08 Marks)**

   (b) Find the pedal equation of the following curve : $r(1 + \cos\theta) = 2a$. **(06 Marks)**

   (c) If $u = f(x - y, y - z, z - x)$, show that $u_x + u_y + u_z = 0$ **(06 Marks)**

Module-III

(a) Obtain a reduction formula for $\int_0^{\pi/2} \sin^n x dx, (n > 0)$. **(08 Marks)**

(b) Evaluate: $\int_0^{\infty} \frac{x^2 dx}{(1+x^2)^3}$ **(06 Marks)**

(c) Evaluate: $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z} (x+y+z) dydxdz$ **(06 Marks)**

OR

6. (a) Obtain a reduction formula for $\int_0^{\pi/2} \cos^n x dx, (n > 0)$ **(08 Marks)**

(b) Evaluate : $\int_0^{2a} x^3 \sqrt{2ax - x^2} dx$ **(06 Marks)**

(c) Evaluate $\iint_R xydxdy$ where R is the first quadrant of the circle $x^2 + y^2 = a^2, x \geq 0, y \geq 0$. **(06 Marks)**

Module-IV

7. (a) A particle moves along a curve $x = e^{-t}, y = 2\cos 3t, z = 2\sin 3t$ where t is the time variable. Determine the components of velocity and acceleration vectors at $t = 0$ in the direction of $\vec{i} + \vec{j} + \vec{k}$. **(08 Marks)**

(b) Find the directional derivative of $\phi = xy^2 + yz^3$ at the point $(2,1,-1)$ in the direction of the vector $\vec{i} + 2\vec{j} + 2\vec{k}$. **(06 Marks)**

(c) Find the values of the constants a, b, c such that $\vec{F} = (x + y + az)\vec{i} + (bx + 2y - z)\vec{j} + (x + cy + 2z)\vec{k}$ is irrotational. **(06 Marks)**

OR

8. (a) If $\vec{F} = (x + y + z)\vec{i} + \vec{j} - (x + y)\vec{k}$, show that $\vec{F} \times curl\vec{F} = 0$ **(08 Marks)**

(b) If $\phi(x, y, z) = x^3 + y^3 + z^3 - 3xyz$, find $\nabla\phi$ & $|\nabla\phi|$ at $(1,-1,2)$ **(06 Marks)**

(c) Show that vector field $\vec{F} = [(xi + yj)/(x^2 + y^2)]$ is solenoidal. **(06 Marks)**

Module-V

9. (a) Solve: $(x^2 + 2xy - y^2)dx + (y^2 + 2xy - x^2)dy = 0$ **(08 Marks)**

(b) Solve: $[y(1 + 1/x) + \cos y]dx + [x + \log x - x\sin y]dy = 0$ **(06 Marks)**

(c) Solve: $(1 + y^2)dx = (\tan^{-1} y - x)dy$ **(06 Marks)**

OR

10. (a) Solve: $(x + 2y - 3)dx - (2x + y - 3)dy = 0$ **(08 Marks)**

(b) Solve: $\left[y^2 e^{xy^2} + 4x^3\right]dx + \left[2xye^{xy^2} - 3y^2\right]dy = 0$ **(06 Marks)**

(c) Solve: $(x^3 \cos^2 y - x\sin 2y)dx = dy$ **(06 Marks)**

*****