Model Question Paper- I with effect from 2020-21 (CBCS Scheme)

USN | | | | | | | | | | | **18MAT653**

Sixth Semester B.E.(CBCS) Examination

ADVANCED LINEAR ALGEBRA

(Open Elective)

Time: 3 Hrs Max.Marks: 100

Note: Answer any FIVE full questions, choosing at least ONE question from each module.

Module-I

1.(a) Test for consistency and solve the system of linear equations : **(07 Marks)**
$x + 2y + z = 3$, $2x + 3y + 3z = 10$ and $3x - y + 2z = 13$.

(b) Solve the following system of linear equations by LU decomposition method:
$3x + 4y + 5z = 18,$ $2x - y + 8z = 13$ and $5x - 2y + 7z = 20$. **(07 Marks)**

(c) Define a subspace. Prove that the intersection of two subspaces of a vector space V(F) is a subspace of V(F). **(06 Marks)**

OR

2. (a) Investigate the values of λ and μ such that the system of equations: **(07 Marks)**
x +y +z = 6 , x +2y +3z = 10 and x +2y + λ z = μ may have
i) unique solution , ii) infinite solution and iii) no solution.

(b) Find the co-ordinate vector of (10 ,5, 0) relative to the vectors (1, -1 ,1) , (0 ,1, 2) and (3 ,0, -1) . **(07 Marks)**

(c) Prove that the set W = {(x, y, z) / x - 3y + 4z = 0}of the vector spaceV_3(R) is a subspace of V_3(R). **(06 Marks)**

Module-II

3.(a) Prove that T: $R^3 \rightarrow R^3$ be defined by T(a, b, c) = (3a , a – b , 2a +b + c) is a linear transformation. **(07 Marks)**

(b) Let T: $R^2 \rightarrow R^2$ be a linear transformation such that T(2,3) = (1,0) and T(3,2) = (1,-1) . **(07 Marks)**
Find the matrix representation of T.

(c) Let T: $R^3 \rightarrow R^3$ be a linear transformation defined by T(x, y, z) = (x+2y - z , y + z, , x + y-2z).
Find the basis and dimension of i) image of T and ii) kernel of T. **(06 Marks)**

OR

4.(a) Find the matrix of the linear transformation $T:V_2(R) \rightarrow V_3(R)$ defined by $T(x, y) = (x + y , x, 3x - y)$ with respect to $B_1 = \{(1,1), (3,1)\}$, $B_2 = \{(1,1,1) ,(1,1,1) ,(1,0,0) \}$. **(07 Marks)**

(b) Let $T: V \rightarrow W$ be a linear transformation defined by $T(x, y, z) = (x + y , x - y, 2x + z)$. Find the range, null space , rank and nullity of T. **(07 Marks)**

(c) Let $T: V \rightarrow W$ be a linear transformation. Then prove that R(T) is a subspace of W. **(06 Marks)**

Module-III

5.(a) Define an inner product space. If V is an inner product space, then for any vectors α, β in V prove that $\| \alpha +\beta \| \leq \|\alpha \| + \| \beta \|$. **(07Marks)**

(b) Prove that an orthogonal set of non zero vectors is linearly independent. **(07 Marks)**

(c) Apply the Gram-Schmidt orthogonalization process to find an orthonormal basis for the subspace of R^4 spanned by the vectors $v_1 = (1,1,1,1)$, $v_2 = (1,2,4,5)$, $v_3 = (1,-3,-4,-2)$. **(06 Marks)**

OR

6 (a)Find the QR decomposition of the matrix **(07 Marks)**

$$A=\begin{bmatrix}1 & 0 & 0\\1 & 1 & 0\\1 & 1 & 1\\1 & 1 & 1\end{bmatrix}$$

(b) Prove that every finite dimensional inner product space has an orthonormal basis. **(07 Marks)**

(c) If V is an inner product space, then for any vectors α , β in V and any scalar c , prove that

i) $|| c\alpha || = c|| \alpha ||$ ii) $|| \langle\alpha , \beta \rangle || = || \alpha || \, || \beta||$. **(06 Marks)**

Module-IV

7.(a) Find the singular value decomposition of A = $\begin{pmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{pmatrix}$ **(07 marks)**

(b) Find the minimum and maximum values of Q(x) = $9x^2 + 4y^2 + 3z^2$ subject to the constraint $X^{\mathbf{T}} X = 1$.

(07 marks)

(c) Diagonalize the matrix A, given that A= $\begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$ **(06marks)**

OR

8.(a) Find the singular value decomposition of A = $\begin{pmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{pmatrix}$ **(07 marks)**

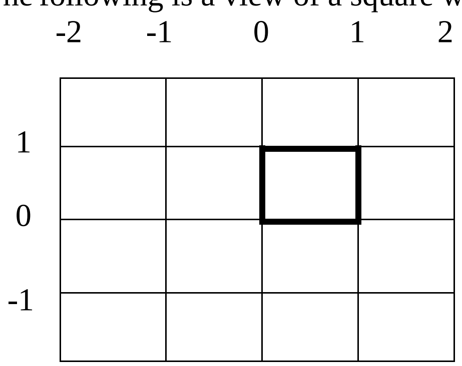
(b) Orthogonally diagonalize the matrix A = $\begin{pmatrix} 3 & -2 & 4 \\ -2 & 6 & 2 \\ 4 & 2 & 3 \end{pmatrix}$ **(07 marks)**

(c) Make a change of variable X = PY that transforms the quadratic form $x_1^2 - 8x_1^2 x_2^2 - 5x_2^2$ in to a quadratic form with no cross product term. **(06 marks)**

Module-V

9.The following is a view of a square with vertices *(*0, 0, 0*)*, *(*1, 0, 0*)*,*(*1, 1, 0*)*, and *(* 0, 1, 0*)*.

(20 marks)



(a) What is the coordinate matrix of view ?

(b) Find the coordinate matrix of view after it is scaled by a factor 1.5 in the *x*-direction and 0.5 in the *y*-direction. Draw the resultant scaled view.

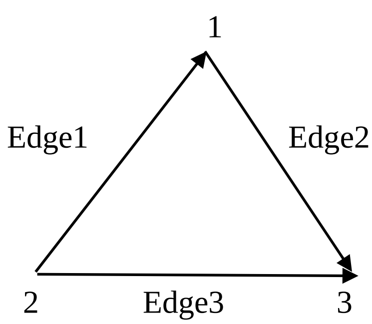
(c) What is the coordinate matrix of view after it is translated by the vector $(-2, -3, 1)^T$?. Sketch the translated view.

(d) Find the coordinate matrix of view after it is rotated through an angle of -30° about the *z*-axis. Draw the rotated view.

OR

10.Write down the 3×3 matrix for the following triangle graph. The first row has -1 in column 1 and +1 in column 2. What vectors (x_1 ,x_2 , x_3) are in its null space .How do you know that (1 ,0,0) is not in its row space.

(20 marks)



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