Model Question Paper- II with effect from 2020-21 (CBCS Scheme)

USN | | | | | | | | | | | **18MAT653**

Sixth Semester B.E.(CBCS) Examination

ADVANCED LINEAR ALGEBRA

(Open Elective)

Time: 3 Hrs Max.Marks: 100

Note: Answer any FIVE full questions, choosing at least ONE question from each module.

Module-I

1.(a) Determiner the values of 'k' such that the system of linear equations : $x + y + z = 1$, $x + 2y + 4z = k$ and $x + 4y + 10z = k^2$ is consistent and hence solve. **(07 Marks)**

(b) Let W be the subspace of R^5 spanned by S = { (1,2,-1,3,4),(2,4,-2,6,8),(1,3,2,2,6), (1,4,5,1,8),(2,7,4,4,9)}. Find a subset of S that form a basis of W. **(07 Marks)**

(c) Prove that W = {(x ,y, z) / x, y,z ∈ F and 15x + 4y+ z = 0} is a subspace of V(F). **(06 Marks)**

OR

2.(a) Solve the following system of linear equations by LU decomposition method : $5x + y + 3z = 20,$ $2x + 5y + 2z = 18$ and $3x + 2y + z = 14$. **(07 Marks)**

(b) Find the co-ordinate vector of (10 ,5, 0) relative to the vectors (1, -1 ,1), (0 ,1, 2) and (3 ,0, -1). **(07 Marks)**

(c) Prove that the set S = {(1 ,2 ,1), (2 ,1, 0), (1, -1, 2) }forms a basis for V_3(F). **(06 Marks)**

Module-II

3.(a) Define a linear transformation. Let T:$R^3 \rightarrow R^2$ be defined by T(x,y,z) =(3x -2y + z ,x – 3y -2z). Prove that T is a linear transformation. (**07 Marks)**

(b) Let T:$R^3 \rightarrow R^2$ be a linear transformation defined by T(x,y,z) = (x + y , y + z).Find a basis and dimension of Rank(T) and Nullity(T). (**07 Marks)**

(c) Let T: $R^3 \rightarrow R^3$ be a linear transformation defined by T(x,y,z) = (2y + z ,x – 4y ,3x). Find matrix representation of T relative to the basis (1,0,1) ,(-1,2,1), (2,1,1). **(06 Marks)**

OR

4.(a) Let $T:R^3 \rightarrow R^3$ be a linear operator defined by $T(x, y, z) = (3x , x - y , 2x +y +z)$. **(07 Marks)**
Prove that $(T^2 - I)(T - 3I) = 0$.

(b) If $T: R^3 \rightarrow R^2$ is a linear transformation defined by $T(x, y, z) = (y- x , y - z)$,find the range, null space , rank and nullity of T. **(07 Marks)**

(c) Let $T: V \rightarrow W$ be a linear transformation. Then prove that N(T) is a subspace of V. **(06 Marks)**

Module-III

5.(a) Find the QR decomposition of the matrix : **(07Marks)**

$$A = \begin{pmatrix} -4 & 4 & 2 \\ 4 & -4 & 1 \\ 2 & 1 & 0 \end{pmatrix}$$

(b)Apply the Gram-Schmidt orthogonalization process to find an orthonormal basis for the subspace of R^4 spanned by the vectors $v_1 = (1,1,1,1)$, $v_2 = (0, 1,1,1)$, $v_3 = (0,0,1,1)$. **(07Marks)**

(c) If W is a subspace of a real inner product space V, prove that $W^{\perp}$ is a subspace of V. **(06 Marks)**

OR

6(a) Let $[-1, 1]$ have the integral inner product $\langle p(x), q(x) \rangle = \int_{-1}^{1} p(x)\, q(x)\, dx$ and let $p(x) = x^2 - x$ and $q(x) = x + 1$. Find i) $\langle p(x), q(x) \rangle$,ii) $\|p(x)\|$ & $\|q(x)\|$ and iii) the cosine of the angle between the vectors $p(x)$ and $q(x)$. **(07 Marks)**

(b) Find an orthonormal basis for the vector space $V_3(R)$ by applying the Gram-Schmidt orthogonalization process to the vectors (3,0,4) , (-1,0,7) and (2,9,11). **(07 Marks)**

(c) Let $u_1 = (2, 5, -1)^T$, $u_2 = (-2 ,1, 1)^T$ and $y = (1, 2, 3)^T$ .If W = span{ u_1 ,u_2},write y as the sum of a vector in W and a vector orthogonal to W.

(06 Marks)

Module-IV

7.(a)Find the singular value decomposition of the matrix $A = \begin{pmatrix} 4 & -2 \\ 2 & -1 \\ 0 & 0 \end{pmatrix}$ **(07 marks)**

(b) Diagonalize the matrix A, given that $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$. Hence find A^4. **(07 marks)**

(c) Find the minimum and maximum values of $Q(x) = 2x^2 + 4y^2 + z^2$ subject to the constraint $X^TX = 1$.

(06marks)

OR

8.(a) Find the singular value decomposition of the matrix $A = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{pmatrix}$ **(07 marks)**

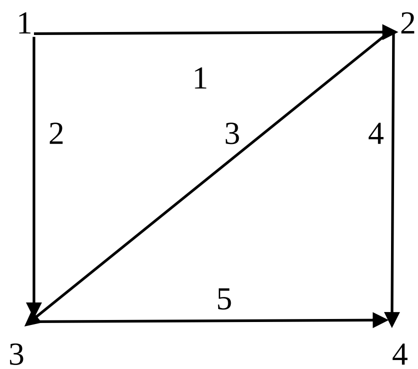
(b) Orthogonally diagonalize the matrix $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ **(07 marks)**

(c) Make a change of variable X = PY that transforms the quadratic form $x_1^2 + 10x_1^2\, x_2^2 + x_2^2$ into a quadratic form with no cross product term. **(06 marks)**

Module-V

9. For the following graph , i) write down the 5 × 4 incidence matrix A with two loops, ii)find one solution to A**x** = 0 and two solutions to $A^T\mathbf{y} = 0$ and iii) reduce matrix A to its echelon form .

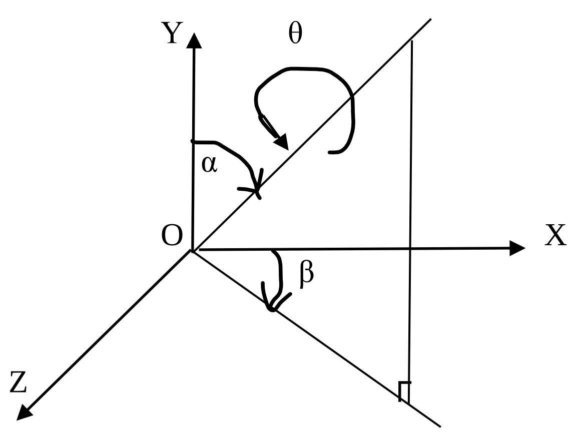
(20 marks)



OR

10. The following is a view with a coordinate matrix P to be rotated through an angle θ about an axis through the origin and specified by two angles α and β. If P^1 is the coordinate matrix of the rotated view, find rotation matrices R_1, R_2, R_3, R_4, and R_5 such that $P^1 = R_5R_4R_3R_2R_1P$.

(20 marks)



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