Model Question Paper-1 with effect from 2018-19
(CBCS Scheme)

USN | | | | | | | | | | | **18MAT21**

Second Semester B.E. Degree Examination
Advanced Calculus and Numerical Methods

(Common to all Branches)

Time: 3 Hrs Max.Marks: 100

Note: Answer any FIVE full questions, choosing at least ONE question from each module.

Module-1

1. (a) Find the angle between the surfaces $x^2 + y^2 - z^2 = 4$ and $z = x^2 + y^2 - 13$ at $(2,1,2)$ **(06 Marks)**

   (b) If $\vec{F} = \nabla(xy^3z^2)$, find $div\vec{F}$ and $curl\vec{F}$ at the point $(1,-1,1)$. **(07 Marks)**

   (c) Find the value of a, b, c such that $\vec{F} = (axy + bz^3)\vec{i} + (3x^2 - cz)\vec{j} + (3xz^2 - y)\vec{k}$ is a conservative force field. Hence find the scalar potential ϕ such that $\vec{F} = \nabla\phi$. **(07 Marks)**

OR

2. (a) Use Green's theorem to find the area between the parabolas $x^2 = 4y$ and $y^2 = 4x$. **(06 Marks)**

   (b) Using Gauss divergence theorem, evaluate $\iint_s \vec{F} \cdot \hat{n}dS$ over the entire surface of the region above xy-plane bounded by the cone $z^2 = x^2 + y^2$ and the plane $z = 4$, where $\vec{F} = 4xz\vec{i} + xyz^2\vec{j} + 3z\vec{k}$. **(07 Marks)**

   (c) Find the work done by the force $\vec{F} = 3x^2\vec{i} + (2xz - y)\vec{j} + z\vec{k}$, when it moves a particle from the point $t = 0$ to $t = 2$ along the curve $x = t, y = t^2/4, z = 3t^3/8$. **(07 Marks)**

Module-2

3. (a) Solve: $(D^3 + D^2 - 4D - 4)y = 3e^{-x} - 4x - 6$, where $D = \frac{d}{dx}$. **(06 Marks)**

   (b) Solve: $\frac{d^2y}{dx^2} + y = \frac{1}{1 + \sin x}$, using the method of variation of parameters. **(07 Marks)**

   (c) Solve: $(x^2D^2 - 3xD + 4)y = (1 + x)^2$, where $D = \frac{d}{dx}$. **(07 Marks)**

OR

4. (a) Solve: $(D^3 + 8)y = x^4 + 2x + 1$, where $D = \frac{d}{dx}$. **(06 Marks)**

   (b) Solve: $(3x + 2)^2 \frac{d^2y}{dx^2} + 3(3x + 2)\frac{dy}{dx} - 36y = 8x^2 + 4x + 1$ **(07 Marks)**

(c) The differential equation of the displacement *x(t)* of a spring fixed at the upper end and a weight at its lower end is given by $10\frac{d^2x}{dt^2}+\frac{dx}{dt}+200x=0$. The weight is pulled down 0.25 cm, below the equilibrium position and then released. Find the expression for the displacement of the weight from its equilibrium position at any time t during its first upward motion. **(07 Marks)**

Module-3

5. (a) Form the partial differential equation by eliminating the arbitrary constants from $(x-a)^2+(y-b)^2+z^2=c^2$ **(06 Marks)**

(b) Solve $\frac{\partial^2 z}{\partial y^2}=z$, given that when $y=0, z=e^x$ and $z=e^{-x}$ **(07 Marks)**

(c) Derive one-dimensional wave equation in the standard form. **(07 Marks)**

OR

6. (a) Form the partial differential equation by eliminating the arbitrary function from $f\left(x^2+y^2, z-xy\right)=0$ **(06 Marks)**

(b) Solve: $\left(x^2-yz\right)p+\left(y^2-zx\right)q=z^2-xy$ **(07 Marks)**

(c) Solve one dimensional heat equation, using the method of separation of variables. **(07 Marks)**

Module-4

7. (a) Test for the convergence or divergence of the series : $\sum_{n=1}^{\infty}\frac{n!}{\left(n^n\right)^2}$ **(06 Marks)**

(b) Solve Bessel's differential equation leading to $J_n(x)$. **(07 Marks)**

(c) Express $f(x)=x^4+3x^3-x^2+5x-2$ in terms of Legendre polynomials. **(07 Marks)**

OR

8. (a) Test for the convergence or divergence of the series : $\sum_{n=1}^{\infty}\frac{n^2}{3^n}$ **(06 Marks)**

(b) If α and β are two distinct roots of $J_n(x)=0$, prove that $\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx=0$ if $\alpha\neq\beta$. **(07 Marks)**

(c) Use Rodrigues's formula to show that $P_4(\cos\theta)=\frac{1}{8}(35\cos 4\theta+20\cos 2\theta+9)$ **(07 Marks)**

Module-5

9. (a) Find a real root of the equation $x\sin x+\cos x=0$, near $x=\pi$ correct to four decimal places, using Newton- Raphson method. **(06 Marks)**

(b) Use an appropriate interpolation formula to compute $f(2.18)$ using the following data: **(07 Marks)**

x	1.7	1.8	1.9	2.0	2.1	2.2
f(x)	5.474	6.050	6.686	7.389	8.166	9.025

(c) Use Weddle's rule to evaluate $\int_{-\pi/2}^{\pi/2}\cos x\,dx$, by dividing $[-\pi/2,\pi/2]$ into six equal parts. **(07 Marks)**

OR

10. (a) Find a real root of $x\log_{10}x-1.2=0$, correct to three decimal places lying in the interval *(2,3)*, using Regula-Falsi method. **(06 Marks)**

(b) Using Lagrange's interpolation formula to fit a polynomial for the following data: **(07 Marks)**

x	2	10	17
y	1	3	4

(c) Using Simpson's $(3/8)^{th}$ rule, evaluate $\int_0^3 \frac{dx}{(1+x)^2}$ taking 4 equidistant ordinates. **(07 Marks)**

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