Model Question Paper-2 with effect from 2018-19
(CBCS Scheme)

USN | | | | | | | | | | | **18MAT21**

Second Semester B.E. Degree Examination
Advanced Calculus and Numerical Methods

(Common to all Branches)

Time: 3 Hrs Max.Marks: 100

Note: Answer any FIVE full questions, choosing at least ONE question from each module.

Module-1

1. (a) Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at $(2,-1.2)$ along $2\vec{i} - 3\vec{j} + 6\vec{k}$. **(06 Marks)**

(b) Find the values of a and b such that the surfaces $ax^2 - byz = (a+2)x$ and $4x^2y + z^3 = 4$ are orthogonal at the point $(1,-1,2)$. **(07 Marks)**

(c) Show that $\vec{F} = \dfrac{x\vec{i} + y\vec{j}}{x^2 + y^2}$ is both solenoidal and irrotational. **(07 Marks)**

OR

2. (a) Use Green's theorem to evaluate $\int_C (x^2 + y^2)dx + 3x^2y\,dy$, where C is the circle $x^2 + y^2 = 4$, traced in the positive sense. **(06 Marks)**

(b) Using Stoke's theorem, evaluate $\oint_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = y\vec{i} + z\vec{j} + x\vec{k}$ and C is the boundary of upper half of the sphere $x^2 + y^2 + z^2 = 1$. **(07 Marks)**

(c) Find the flux of $\vec{F} = \vec{i} - \vec{j} + xyz\vec{k}$, through the circular region S obtained by cutting the sphere $x^2 + y^2 + z^2 = a^2$ with the plane $y = x$. **(07 Marks)**

Module-2

3. (a) Solve: $(D^2 + 4)y = x^2 + \cos 2x$, where $D = \dfrac{d}{dx}$ **(06 Marks)**

(b) $\dfrac{d^2y}{dx^2} + y = \sec x \tan x$, using the method of variation of parameters. **(07 Marks)**

(c) Solve: $(x^2D^2 + xD + 9)y = 3x^2 + \sin(3\log x)$, where $D = \dfrac{d}{dx}$ **(07 Marks)**

OR

4. (a) Solve: $\frac{d^3y}{dx^3}+2\frac{d^2y}{dx^2}+\frac{dy}{dx}=x^3$ **(06 Marks)**

(b) Solve: $(3x+2)^2\frac{d^2y}{dx^2}+3(3x+2)\frac{dy}{dx}-36y=8x^2+4x+1$ **(07 Marks)**

(c) In an L-C-R circuit, the charge q on a plate of a condenser is given by $L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{C}=E\sin pt$. The circuit is tuned to resonance so that $p^2=1/LC$. If initially the current i and the charge q be zero, show that for small values of R/L, the current in the circuit at time t is given by $(Et/2L)\sin pt$. **(07 Marks)**

Module-3

5. (a) Form the partial differential equation by eliminating the arbitrary functions from $z=yf(x)+x\varphi(y)$ **(06 Marks)**

(b) Solve $\frac{\partial^2 z}{\partial x\partial y}=\sin x\sin y$, for which $\frac{\partial z}{\partial y}=-2\sin y$ when $x=0$ and $z=0$ when y is odd. **(07 Marks)**

(c) Solve one dimensional wave equation, using the method of separation of variables. **(07 Marks)**

OR

6. (a) Form the partial differential equation by eliminating the arbitrary constants from $2z=\frac{x^2}{a^2}+\frac{y^2}{b^2}$. **(06 Marks)**

(b) Solve: $(x^2-y^2-z^2)p+2xyq=2xz$ **(07 Marks)**

(c) Derive one-dimensional heat equation in the standard form. **(07 Marks)**

Module-4

7. (a) Use Rodrigues's formula to show that $P_3(\cos\theta)=\frac{1}{8}(3\cos\theta+5\cos 3\theta)$ **(06 Marks)**

(b) Solve Legendre's differential equation leading to $P_n(x)$. **(07 Marks)**

(c) Discuss the nature of the series : $\frac{1}{2}+\left(\frac{2}{3}\right)x+\left(\frac{3}{4}\right)^2x^2+\left(\frac{4}{5}\right)^3x^3+\ldots$ **(07 Marks)**

OR

8. (a) Express $f(x)=x^3+2x^2-x-3$ in terms of Legendre polynomials. **(06 Marks)**

(b) With usual notation, show that (i) $J_{1/2}(x)=\sqrt{(2/\pi x)}\sin x$ (ii) $J_{-1/2}(x)=\sqrt{(2/\pi x)}\cos x$. **(07 Marks)**

(c) Test for the convergence or divergence of the series : $\frac{1}{2\sqrt{1}}+\frac{x^2}{3\sqrt{2}}+\frac{x^4}{4\sqrt{3}}+\frac{x^6}{5\sqrt{4}}+\ldots$ **(07 Marks)**

Module-5

9. (a) Find a real root of $xe^x - \cos x = 0$, correct to three decimal places lying in the interval *(0.5,0.6)*, using Regula-Falsi method. **(06 Marks)**

(b) Using divided difference formula, fit a polynomial for the following data: **(07 Marks)**

x	2	4	5	6	8	10
y	10	96	196	350	868	1746

(c) Evaluate $\int_0^{\pi/2} \sqrt{\sin x}\, dx$ using Simpson's $(1/3)^{rd}$ rule, taking 10 equal parts. **(07 Marks)**

OR

10. (a) Find a real root of the equation $x^3 + x^2 + 3x + 4 = 0$ near $x = -1$ correct to four decimal places, using Newton- Raphson method. **(06 Marks)**

(b) Use an appropriate interpolation formula to compute $f(42)$ using the following data: **(07 Marks)**

x	40	50	60	70	80	90
f(x)	184	204	226	250	276	304

(c) Use Weddle's rule to evaluate $\int_0^1 \frac{xdx}{1+x^2}$, by taking seven ordinates. **(07 Marks)**

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