

# Model Question Paper-2 with effect from 2019-20 (CBCS Scheme)

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18MATDIP31

## Third Semester B.E. Degree Examination Additional Mathematics-I

(Common to all Programmes)

Time: 3 Hrs

Max.Marks: 100

**Note: Answer any FIVE full questions, choosing at least ONE question from each module.**

### Module-I

1. (a) Show that  $\left(\frac{1 + \sin \theta + i \cos \theta}{1 + \sin \theta - i \cos \theta}\right)^n = \cos\left(\frac{n\pi}{2} - n\theta\right) + i \sin\left(\frac{n\pi}{2} - n\theta\right)$ . (08 Marks)
- (b) Express  $\sqrt{7} + 9i$  in the polar form and hence find its modulus and amplitude. (06 Marks)
- (c) Find the real part of  $\frac{1}{1 + \cos \theta + i \sin \theta}$  (06 Marks)

**OR**

2. (a) If  $\vec{A} = \vec{i} + 2\vec{j} + 3\vec{k}$ ,  $\vec{B} = -\vec{i} + 2\vec{j} + \vec{k}$  and  $\vec{C} = 3\vec{i} + \vec{j}$ , find  $p$  such that  $\vec{A} + p\vec{B}$  is perpendicular to  $\vec{C}$ . (08 Marks)
- (b) Find the area of the parallelogram whose adjacent sides are the vectors  $\vec{A} = 2\vec{i} + 4\vec{j} - 5\vec{k}$  and  $\vec{B} = \vec{i} + 2\vec{j} + 3\vec{k}$ . (06 Marks)
- (c) If  $\vec{A} = 4\vec{i} + 3\vec{j} + \vec{k}$  and  $\vec{B} = 2\vec{i} - \vec{j} + 2\vec{k}$ , find a unit vector  $N$  perpendicular to both  $\vec{A}$  and  $\vec{B}$  such that  $\vec{A}$ ,  $\vec{B}$  and  $N$  form a right handed system. (06 Marks)

### Module-II

3. (a) Obtain the Maclaurin's series expansion of  $e^{m \cos^{-1} x}$  up to the terms containing  $x^5$ . (08 Marks)
- (b) Prove that  $xu_x + yu_y = 3$ , where  $u = \log\left[\frac{(x^4 + y^4)}{(x + y)}\right]$ , using Euler's theorem, (06 Marks)
- (c) If  $u = f(x - y, y - z, z - x)$ , show that  $u_x + u_y + u_z = 0$  (06 Marks)

**OR**

4. (a) Prove that  $\log(\sec x + \tan x) = x + \frac{x^3}{6} + \frac{x^5}{24} + \dots$ , by using Maclaurin's series notion. (08 Marks)
- (b) Using Euler's theorem, prove that  $xu_x + yu_y = -2 \cot u$ , where  $u = \cos^{-1}\left[\frac{(x^3 + y^3)}{(x + y)}\right]$ . (06 Marks)
- (c) If  $u = 2xy, v = x^2 - y^2$  &  $x = r \cos \theta, y = r \sin \theta$ , compute  $\frac{\partial(u, v)}{\partial(r, \theta)}$ . (06 Marks)

### Module-III

5. (a) A particle moves on the curves  $x = 1 - t^3, y = 1 + t^2, z = 2t - 5$  where  $t$  is the time variable. Determine the components of velocity and acceleration vectors at  $t = 1$  in the direction of  $\vec{i} + 2\vec{j} + \vec{k}$ . (08 Marks)

- (b) Find the unit normal to the surface  $xy^3z^2 = 4$  at  $(1, -1, 2)$  (06 Marks)
- (c) Show that the vector field  $\vec{F} = (x + y + z)\vec{i} + (x + 2y - z)\vec{j} + (x - y + 2z)\vec{k}$  is irrotational. (06 Marks)

OR

6. (a) Find  $\text{div}\vec{F}$  and  $\text{curl}\vec{F}$  where  $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$  (08 Marks)
- (b) If  $\vec{F} = (3x^2y - z)\vec{i} + (xz^3 + y^4)\vec{j} - 2x^3z^2\vec{k}$ , find  $\text{grad}(\text{div}\vec{F})$  at  $(2, -1, 0)$  (06 Marks)
- (c) Find the value of 'a' such that vector field  $\vec{F} = (x + 3y)\vec{i} + (y - 2z)\vec{j} + (x + az)\vec{k}$  is solenoidal. (06 Marks)

**Module-IV**

7. (a) Obtain a reduction formula for  $\int_0^{\pi/2} \cos^n x dx, (n > 0)$  (08 Marks)
- (b) Evaluate:  $\int_0^a \frac{x^2 dx}{(1 + x^6)^{7/2}}$ . (06 Marks)
- (c) Evaluate  $\iint_R (x^2 + y^2) dx dy$ , where  $R$  is the region bounded by  $y = x$  &  $y^2 = 4x$  (06 Marks)

OR

8. (a) Obtain a reduction formula for  $\int_0^{\pi/2} \sin^n x dx, (n > 0)$ . (08 Marks)
- (b) Evaluate :  $\int_0^a x\sqrt{ax - x^2} dx$  (06 Marks)
- (c) Evaluate:  $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dx dy dz}{\sqrt{1-x^2-y^2-z^2}}$  (06 Marks)

**Module-V**

9. (a) Solve:  $y(2x - y + 1)dx + x(3x - 4y + 3)dy = 0$  (08 Marks)
- (b) Solve:  $[y^2 e^{xy^2} + 4x^3]dx + [2xye^{xy^2} - 3y^2]dy = 0$  (06 Marks)
- (c) Solve:  $dx + (x - e^{-y} \sec^2 y)dy = 0$  (06 Marks)

OR

10. (a) Solve:  $\tan y dy = (\cos y \cos^2 x - \tan x)dx$  (08 Marks)
- (b) Solve:  $[y(1 + 1/x) + \cos y]dx + [x + \log x - x \sin y]dy = 0$  (06 Marks)
- (c) Solve:  $(1 + x^2)(dy - dx) = 2xydy$  (06 Marks)

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