Model Question Paper-I with effect from 2025

USN | | | | | | | | | | | **1BMATE201**

Second Semester B.E./B.Tech. Degree Examination
Calculus, Laplace Transforms and Numerical Techniques

TIME: 03Hours **Max.Marks:100**

Note: 1. *Answer any **FIVE** full questions, choosing at least **ONE** question from each **MODULE***
2. *VTU Formula Hand Book is Permitted*
3. *M: Marks, L: Bloom's level, C: Course outcomes*

		Module-1	**M**	**L**	**C**
Q 1.	**a**	Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dx\,dy\,dz$.	**6**	**L2**	1
	b	Evaluate $\int_0^a \int_0^{\sqrt{a^2-y^2}} y\sqrt{x^2+y^2}\,dx\,dy$ by changing into polar co-ordinates.	**7**	**L2**	1
	c	With usual notations derive the relation between beta and gamma function $\beta(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$	**7**	**L2**	1
		OR			
Q 2.	**a**	Evaluate $\int_0^3 \int_1^{\sqrt{4-y}} \frac{dy\,dx}{\log y}$ by changing the order of integration.	**6**	**L2**	1
	b	Find the area between parabolas $y^2=4ax$ and $x^2=4ay$ using double integration.	**7**	**L2**	1
	c	Prove that $\int_0^{\pi/2}\frac{d\theta}{\sin\theta}\times\int_0^{\pi/2}\sin\theta\,d\theta=\pi$	**7**	**L2**	1
		Module-2			
Q 3.	**a**	Find the directional derivative of $f(x,y,z)=xy^3+yz^3$at the point $(2,-1,1)$ in the direction of the vector $\vec{i}+2\vec{j}+2\vec{k}$.	**6**	**L2**	1
	b	Show that $\vec{F}=(2xy^2+yz)\vec{i}+(2x^2y+xz+2yz^2)\vec{j}+(2y^2z+xy)\vec{k}$ is a conservative force field (Irrotational). Find its scalar function $\emptyset$ such that $\vec{F}=\nabla\emptyset$.	**7**	**L2**	1
	c	Applying Green's theorem, evaluate $\int_C [(y-sinx)dx+cosxdy]$ where C is the plane triangle enclosed by the lines $y=0,\ x=\frac{\pi}{2}\ and\ y=\frac{2}{\pi}x$	**7**	**L2**	1
		OR			
Q 4.	**a**	Find the angle between the surfaces $x^2+y^2+z^2=9\,,x^2+y^2-z=3$ at the point (2, -1, 2).	**6**	**L2**	1
	b	Show that $\vec{F}=\frac{x\vec{i}+y\vec{j}}{x^2+y^2}$ is both Solenoidal and Irrotational.	**7**	**L2**	1
	c	Applying Stoke's theorem, evaluate $\int_C [(x+y)dx+(2x-z)dy+(y+z)dz]$ where C is the boundary of the triangle with vertices (2,0,0), (0,3,0), (0,0,6).	**7**	**L2**	1
		Module-3			
Q 5.	**a**	Find the approximate root of the equation tanx + tanhx = 0 in (2,3) by applying Regula-Falsi method.	**6**	**L2**	2

<table>
<tr><td></td><td>b</td><td>Apply Newton divided difference formula to find f(4) given the data.<table><tr><td>x</td><td>0</td><td>2</td><td>3</td><td>6</td></tr><tr><td>$f(x)$</td><td>-4</td><td>2</td><td>14</td><td>158</td></tr></table></td><td>7</td><td>L2</td><td>2</td></tr>
<tr><td></td><td>c</td><td>Evaluate $\int_4^{5.2} \log x \, dx$ using Simpson's $3/8$ th & $1/3$ rd rule by taking 6 equal strips.</td><td>7</td><td>L2</td><td>2</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td rowspan="3">Q 6.</td><td>a</td><td>Find an approximate root of the equation, $x \log_{10} x = 1.2$ near 2.5 by applying Newton Raphson, method</td><td>6</td><td>L2</td><td>2</td></tr>
<tr><td>b</td><td>Use appropriate Newton's interpolation formula to fit a polynomial of degree three which takes the following values<table><tr><td>x</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>y</td><td>6</td><td>24</td><td>60</td><td>120</td></tr></table></td><td>7</td><td>L3</td><td>2</td></tr>
<tr><td>c</td><td>By applying Simpson's $1/3$ rd rule, evaluate $\int_0^5 \frac{dx}{4x+5}$ with 11 ordinates and hence findlog 5</td><td>7</td><td>L2</td><td>2</td></tr>
<tr><td colspan="6">Module-4</td></tr>
<tr><td rowspan="3">Q 7.</td><td>a</td><td>Apply Taylor's series method solve $\frac{dy}{dx} = 3x + y^2$ with $y(0) = 1$ and hence find $y(0.1)$ and consider upto 4^{th} degree.</td><td>6</td><td>L2</td><td>2</td></tr>
<tr><td>b</td><td>Solve $\frac{dy}{dx} = \frac{y-x}{y+x}$, with $y(0) = 1$ and hence find $y(0.1)$ by taking one steps using Runge-Kutta method of fourth order.</td><td>7</td><td>L2</td><td>2</td></tr>
<tr><td>c</td><td>Apply Milne's method to solve $\frac{dy}{dx} = x - y^2$ with the following data $y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$, $y(0.6) = 0.1762$, compute y at x=0.8,</td><td>7</td><td>L2</td><td>2</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td rowspan="3">Q 8.</td><td>a</td><td>If $y' + y + 2x = 0$, $y(0) = -1$ then find $y(0.1)$ by using Taylor's series method. Consider upto third order derivative terms.</td><td>6</td><td>L2</td><td>2</td></tr>
<tr><td>b</td><td>Find $y(0.2)$by using modified Euler's method. Given that $y' = x + y$, $y(0) = 1$. Take h=0.1 and carry out two modifications at each step.</td><td>7</td><td>L2</td><td>2</td></tr>
<tr><td>c</td><td>Apply Adam-Bashforth method to find $y(0.4)$ from $y' = \frac{xy}{2}$, $y(0) = 1$, $y(0.1) = 1.0025$, $y(0.2) = 1.0101$, $y(0.3) = 1.0228$.</td><td>7</td><td>L2</td><td>2</td></tr>
<tr><td colspan="6">Module-5</td></tr>
<tr><td rowspan="3">Q 9.</td><td>a</td><td>Find $L(t^2 e^{-3t} \sin 2t)$</td><td>6</td><td>L2</td><td>3</td></tr>
<tr><td>b</td><td>A periodic function of period 2a is defined by $f(t) = \begin{cases} E & for\ 0 \le t \le a \\ -E & for a < t \le 2a \end{cases}$ where E is a constant. Show that $L\{f(t)\} = \frac{E}{s} \tanh\left(\frac{as}{2}\right)$.</td><td>7</td><td>L2</td><td>3</td></tr>
<tr><td>c</td><td>Express the function $f(t) = \begin{cases} \pi - t & ,0 < t \le \pi \\ \sin t & , \quad t > \pi \end{cases}$ in terms of unit step function and hence find its Laplace transform.</td><td>7</td><td>L2</td><td>3</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td rowspan="3">Q 10.</td><td>a</td><td>Find the inverse Laplace transforms of $\frac{2s^2-6s+5}{s^3-6s^2+11s-6}$</td><td>6</td><td>L2</td><td>3</td></tr>
<tr><td>b</td><td>Find $L^{-1}\left\{ log \sqrt{\frac{s^2+a^2}{s^2+b^2}} \right\}$</td><td>7</td><td>L2</td><td>3</td></tr>
<tr><td>c</td><td>Solve $\frac{d^2y}{dx^2}+6\frac{dy}{dx} + 9y = 12t^2e^{-3t}$ subject to the conditions $y(0) = y'(0) = 0$ by using Laplace transforms.</td><td>7</td><td>L3</td><td>3</td></tr>
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