Model Question Paper-I with effect from 2025

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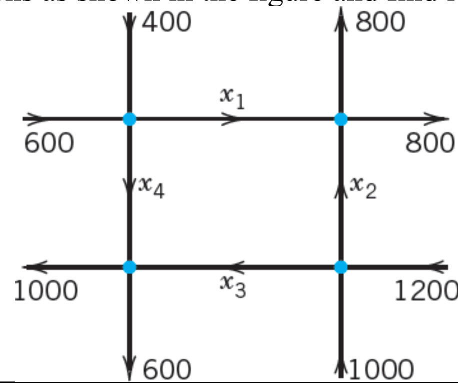
1BMATS101

First Semester B.E./B.Tech. Degree Examination
Calculus & Linear Algebra

TIME: 03Hours **Max.Marks:100**

Note: 1. *Answer any **FIVE** full questions, choosing at least **ONE** question from each **MODULE***
2. *VTU Formula Hand Book is Permitted*
3. *M: Marks, L: Bloom's level, C: Course outcomes*

		Module-1	**M**	**L**	**C**
Q 1.	**a**	Show that $u_x + u_y = u$, if $u = \dfrac{e^{x+y}}{e^x + e^y}$.	**6**	**L2**	1
	b	If $u = x + 3y^2 - z^3, v = 4x^2yz,\ w = 2z^2 - xy$, find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$ at (1, -1,0).	**7**	**L2**	1
	c	Find the extreme values of the function $f(x,y) = xy + \dfrac{a^3}{x} + \dfrac{a^3}{y}$.	**7**	**L3**	1
		OR			
Q 2.	**a**	If $V = f(r,s,t)$ and $r = \dfrac{x}{y}, s = \dfrac{y}{z}, t = \dfrac{z}{x}$ show that $x\dfrac{\partial V}{\partial x} + y\dfrac{\partial V}{\partial y} + z\dfrac{\partial V}{\partial z} = 0$.	**6**	**L2**	1
	B	If $u = \dfrac{2yz}{x},\ v = \dfrac{3zx}{y},\ w = \dfrac{4xy}{z}$ then find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$.	**7**	**L2**	1
	C	Apply Maclaurin's series, to expand $\cos x \cos y$ in powers of x and y up to second-degree terms.	**7**	**L3**	1
		Module-2			
Q 3.	**a**	If $f = x^2yz$ and $g = xy - 3z^2$, calculate $\nabla(\nabla f \cdot \nabla g)$.	**6**	**L2**	1
	b	A vector field is given by $F = (6xy + z^3)\hat{i} + (3x^2 - z)j + (3xz^2 - y)k$. Show that the field is irrotational and hence find its scalar potential.	**7**	**L2**	1
	c	Express the vector $\vec{F} = x\hat{\imath} + 2y\hat{\jmath} + yz\hat{k}$ in spherical polar coordinates.	**7**	**L3**	1
		OR			
Q 4.	**a**	Find the directional derivative of $f(x,y,z) = 4e^{2x-y+z}$ at the point $(1,1,-1)$ in the direction towards the point $(-3, 5, 6)$.	**6**	**L2**	1
	b	Find $div\,\vec{F}$ and $curl\,\vec{F}$, where $F = grad(x^3 + y^3 + z^3 - 3xyz)$.	**7**	**L2**	1
	c	Express the vector $\vec{F} = 2x\hat{\imath} + 3y\hat{\jmath} - z\hat{k}$ in cylindrical polar coordinates.	**7**	**L3**	1
		Module-3			

Q 5.	A	Find the constant b if the rank of $\begin{bmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ b & 2 & 2 & 2 \\ 9 & 9 & b & 3 \end{bmatrix}$ is 3.	**6**	**L2**	2
	b	Find model matrix of $\begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$ and verify its diagonalization.	**7**	**L2**	2
	c	Write the system of linear equations of the traffic flow in the net of one-way street directions as shown in the figure and find its solution.  	**7**	**L3**	2
		OR			
Q 6.	a	Investigate the values of λ and μ so that the equations $\begin{aligned} 2x+3y+5z&=9 \\ 7x+3y-2z&=8 \\ 2x+3y+\lambda z&=\mu \end{aligned}$ have (i) no solution (ii) unique solution (iii) infinite number of solutions.	**6**	**L2**	2
	b	Apply Gauss Jordan method to approximate the solutions of the system $\begin{aligned} 83x+11y-4z&=95 \\ 7x+52y+13z&=104 \\ 3x+8y+29z&=71 \end{aligned}$ by choosing initial solution $(0,0,0)$. Perform four iterations.	**7**	**L2**	2
	C	Determine the eigenvalues and corresponding eigenvectors for the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.	**7**	**L3**	2
		Module-4			
Q 7.	a	Verify whether $v=(1,-2,5)$ in $\mathbb{R}^3$ is a linear combination of the vectors $u_1=(1,1,1)$, $u_2=(1,2,3)$ and $u_3=(2,-1,1)$.	**6**	**L2**	3
	b	Determine whether $W=\{(a,b,c)/a+b+c=0\}$ is a subspace of R^3 or not?	**7**	**L2**	3
	c	Find the basis and dimension of the row space, column space and null space of the matrix $\begin{bmatrix} 1 & -1 & 1 & 3 & 2 \\ 2 & -1 & 1 & 5 & 1 \\ 0 & 1 & -1 & -1 & -3 \end{bmatrix}$	**7**	**L2**	3

OR					
Q 8.	**a**	Find the basis and dimension of the subspace W spanned by $(1,2,3),(2,4,6),(0,1,1)$.	**6**	**L2**	3
	b	Find the inner products $\langle v_1,v_2\rangle,\langle v_1,v_3\rangle$ and $\langle v_2,v_3\rangle$ where $v_1=(1,1,1,1)$, $v_2=(1,2,4,5)$ $v_3=(1,-3,-4,-2)$.	**7**	**L2**	3
	c	Find the coordinates of the vector $v=(1,-3,\ 2)$ with respect to the basis $S=\{(1,\ 1,\ 1),(1,\ 1,\ 0),(1,\ 0,\ 0)\}$.	**7**	**L2**	3
Module-5					
Q 9.	**a**	Verify whether the transformation $T:R^2\to R^2$ which is defined by $T(x,\ y)=(3x+4y,10x-4y+3)$ is linear or not?	**6**	**L2**	3
	b	Prove that the transformation $F:R^2\to R^2$ is singular and find its Kernal if the transformation $F(a,\ b)=(2a-4b,\ 3a-6b)$.	**7**	**L2**	3
	c	Find the rank and nullity of the transformation $T:R^3\to R^3$ defined by $T(x,\ y,\ z)=(x+y,\ x-y,\ 2x+z)$.	**7**	**L2**	3
OR					
Q 10.	**A**	Check whether the transformation $T:V_1(R)\to V_3(R)$ defined by $T(x)=(x,\ x^2,\ x^3)$ is linear or not.	**6**	**L2**	3
	b	Consider the matrix $A=\begin{bmatrix}2&4\\5&6\end{bmatrix}$ which defines a linear operator on $\mathbb{R}^2$. Find the matrix of the linear transformation relative to the basis $S=\{u_1,u_2\}=\left\{\begin{bmatrix}1\\-2\end{bmatrix},\begin{bmatrix}3\\-7\end{bmatrix}\right\}$.	**7**	**L2**	3
		Let F be the linear transformation defined on a vector space R^2 through $F(x,\ y)=(2x+y,\ 3x+2y)$, show that F is invertible and hence find F^{-1}.	**7**	**L2**	3