

Model Question Paper-I (21BS11) with effect from 2021

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First Semester B. Sc Degree Examination CALCULUS AND LINEAR ALGEBRA

TIME: 03 Hours

Max. Marks: 100

Note: Answer any **FIVE** full questions, choosing at least **ONE** question from each module.

Module -1			Marks
Q.01	a	If $y = a \cos(\log x) + b \sin(\log x)$, show that $x^2 y_{n+2} + (2n + 1)xy_{n+1} + (n^2 + 1)y_n = 0$.	07
	b	State and prove Cauchy's mean value theorem.	07
	c	Evaluate $\lim_{x \rightarrow 1} \left[\tan\left(\frac{\pi x}{4}\right) \right]^{\tan\left(\frac{\pi x}{2}\right)}$	06
OR			
Q.02	a	Show that $\frac{d^n}{dx^n} (e^{ax} \sin(bx + c)) = (a^2 + b^2)^{n/2} e^{ax} \sin\left[bx + c + n \tan^{-1}\left(\frac{b}{a}\right)\right]$ and hence find the n^{th} derivative of $e^{3x} \sin(2x - 1)$	07
	b	Verify Lagrange's Mean value theorem for the function $f(x) = (x - 1)(x - 2)(x - 3)$ in $(0, 4)$	07
	c	Using Maclaurin's series, expand $\log(1 + x)$ up to fourth degree term.	06
Module-2			
Q. 03	a	With usual notations prove that $\tan \varphi = r \frac{d\theta}{dr}$	07
	b	Find the curvature of the curve $16y^2 = 4x^4 - x^6$ at the point $(2, 0)$	07
	c	Find the equation of the evolute of an ellipse represented by the parametric equations $x = a \cos t$, $y = b \sin t$	06
OR			
Q.04	a	Find the angle between the curves $r = a(1 + \cos \theta)$ and $r = b(1 - \cos \theta)$	07
	b	With usual notations prove that $\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$	07
	c	Find the radius of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at the point $\left(\frac{a}{4}, \frac{a}{4}\right)$	06
Module-3			
Q. 05	a	If $u = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$, then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$	07
	b	State and prove Euler's theorem on a homogeneous function of degree n	07
	c	Test the function $z = x^2 - xy + y^2 + 3x - 2y + 1$ for maximum and minimum	06
OR			
Q. 06	a	Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$, if $z = u + v^2$, $u = x^2 + \sin y$, $v = \ln(x + y)$	07

	b	Find the Jacobian of the transformation $x = r \sin \theta \cos \varphi, y = r \sin \theta \sin \varphi, z = r \cos \theta$	07
	c	Expand $\tan^{-1}\left(\frac{y}{x}\right)$ in powers of $(x - 1)$ and $(y - 1)$ up to third – degree terms.	06
Module-4			
Q. 07	a	Find the rank of the matrix $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$	06
	b	Solve the system of equations by Gauss elimination method $3x + y + 2z = 3, 2x - 3y - z = -3, x + 2y + z = 4$	07
	c	Using LU decomposition method, solve $x + y + z = 9;$ $x - 2y + 3z = 8;$ $2x + y - z = 3$	07
OR			
Q. 08	a	Test for consistency and solve $5x + 3y + 7z = 4 ;$ $3x + 26y + 2z = 9 ;$ $7x + 2y + 10z = 5$	06
	b	Solve the system of equations by Gauss Jordan method $x + y + z = 15,$ $3x - 3y + z = 1,$ $x + 2y - z = 6$	07
	c	Using Crout's method solve the equations $x + y + z = 7,$ $x + 2y + 3z = 16,$ $x + 3y + 4z = 22$	07
Module-5			
Q. 09	a	Find the Eigen values and the corresponding Eigen vectors of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$	07
	b	State and prove Cayley-Hamilton theorem and verify the theorem for the matrix $A =$ $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$	07
	c	Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ to the canonical form and specify the matrix of transformation.	06
OR			

Q. 10	a	Using Cayley-Hamilton theorem find A^{-1} , if $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$	07
	b	Reduce the matrix $\begin{bmatrix} 4 & 0 & 1 \\ -1 & -6 & -2 \\ 5 & 0 & 0 \end{bmatrix}$ to the diagonal form	07
	c	Show that the linear transformation $y_1 = 2x_1 + x_2 + x_3$, $y_2 = x_1 + x_2 + 2x_3$, $y_3 = x_1 - 2x_3$ is regular. Find the inverse transformation.	06

Table showing the Bloom's Taxonomy Level, Course Outcome and Program Outcome				
Question		Bloom's Taxonomy Level attached	Course Outcome	Program Outcome
Q.1	(a)	L1	CO 01	
	(b)	L2	CO 01	
	(c)	L3	CO 01	
Q.2	(a)	L1	CO 01	
	(b)	L2	CO 01	
	(c)	L3	CO 01	
Q.3	(a)	L2	CO 02	
	(b)	L2	CO 02	
	(c)	L3	CO 02	
Q.4	(a)	L2	CO 02	
	(b)	L2	CO 02	
	(c)	L3	CO 02	
Q.5	(a)	L2	CO 03	
	(b)	L3	CO 03	
	(c)	L2	CO 03	
Q.6	(a)	L2	CO 03	

	(b)	L3	CO 03	
	(c)	L2	CO 03	
Q.7	(a)	L2	CO 04	
	(b)	L2	CO 04	
	(c)	L2	CO 04	
Q.8	(a)	L2	CO 04	
	(b)	L2	CO 04	
	(c)	L2	CO 04	
Q.9	(a)	L2	CO 06	
	(b)	L3	CO 05	
	(c)	L3	CO 05	
Q.10	(a)	L2	CO 06	
	(b)	L3	CO 05	
	(c)	L3	CO 05	
Lower order thinking skills				
Bloom's Taxonomy Levels	Remembering(knowledge):L ₁	Understanding Comprehension): L ₂	Applying (Application): L ₃	
	Higher order thinking skills			
	Analyzing (Analysis):L ₄	Valuating (Evaluation): L ₅	Creating (Synthesis): L ₆	