

Model Question Paper-II (21BS11) with effect from 2021

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First Semester B.Sc. Degree Examination CALCULUS AND LINEAR ALGEBRA

TIME: 03 Hours

Max. Marks: 100

Note: Answer any **FIVE** full questions, choosing at least **ONE** question from each module.

Module -1			Marks
Q.01	a	Find the n^{th} derivative of (i) $\frac{x+1}{x^2-4}$ (ii) $e^{2x} \cos^2 x \sin x$	07
	b	State and prove Lagrange's mean value theorem.	07
	c	Evaluate (i) $\lim_{x \rightarrow 0} \frac{x - \sin hx}{\sin^3 x}$ (ii) $\lim_{x \rightarrow 1} \left[\frac{2}{x^2-1} - \frac{1}{x-1} \right]$	06
OR			
Q.02	a	If $y = e^{m \sin^{-1} x}$, prove that $(1 - x^2)y_{n+2} - 2nxy_{n+1} - (n^2 + m^2)y_n = 0$	07
	b	Verify Cauchy's Mean value theorem for the function $f(x) = x^3$, $g(x) = x^4$ in $(1, 2)$	07
	c	Expand in powers of $(x + 1)$, the polynomial $x^5 + 2x^4 - x^2 + x + 1$	06
Module-2			
Q. 03	a	Find the angle between the curve $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$	07
	b	Find the evolute of the parabola $y^2 = 4ax$	07
	c	Find the radius of curvature of the curve $x = 3t^2$, $y = 3t - t^3$ at $t = 1$	06
OR			
Q.04	a	Obtain the pedal equation of a curve $r = f(\theta)$ in the form $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4} \left(\frac{dr}{d\theta} \right)^2$	07
	b	With usual notations prove that $\rho = \frac{(r^2 + r_1^2)^{3/2}}{r^2 + 2r_1^2 - rr_2}$	07
	c	Find the coordinates of the centre of curvature of the curve $x = a \cos^3 \theta$, $y = a \sin^3 \theta$	06
Module-3			
Q. 05	a	If $u = x^2 \tan^{-1} \left(\frac{y}{x} \right) - y^2 \tan^{-1} \left(\frac{x}{y} \right)$, then prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{x^2 - y^2}{x^2 + y^2}$	07
	b	Using Euler's theorem show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$, if $u = \tan^{-1} \left(\frac{x^3 + y^3}{x - y} \right)$	07
	c	Test the function $z = xy(a - x - y)$ for maximum and minimum	06
OR			
Q. 06	a	If $H = f(y - z, z - x, x - y)$, prove that $\frac{\partial H}{\partial x} + \frac{\partial H}{\partial y} + \frac{\partial H}{\partial z} = 0$	07

	b	If $x = e^u \cos v, y = e^u \sin v$ find $\frac{\partial(x, y)}{\partial(u, v)}$	07
	c	Expand $e^x \log_e(1 + y)$ in powers of x and y up to terms of the third degree.	06
Module-4			
Q. 07	a	Find the rank of the matrix $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$	06
	b	Show that the system of equations $x + y + z = 4,$ $2x + y - z = 1,$ $x - y + 2z = 2$ is consistent. Hence solve the system by Gauss elimination method	07
	c	Using Crout's method, solve $3x + y + 2z = 3 ; 2x - 3y - z = -3 ; x + 2y + z = 4$	07
OR			
Q. 08	a	Find the values of a and b for which the equations $x + ay + z = 3$ $x + 2y + 2z = b$ $2x + 5y + 3z = 9$ are consistent. When do these equations have a unique solution?	06
	b	Find the value of k for which the homogeneous equations $(k - 1)x + (3k + 1)y + 2kz = 0$ $(k - 1)x + (4k - 2)y + (k + 3)z = 0$ $2x + (3k + 1)y + 3(k - 1)z = 0$ are consistent	07
	c	Using the Jordan method solve the equations $x + 2y + z = 16, 2x + y - z = 5, x + 3y + 4z = 35$	07
Module-5			
Q. 09	a	Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$	07
	b	Using Cayley –Hamilton theorem find A^{-1} and A^4 , if $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$	07
	c	Reduce the matrix $\begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$ to the diagonal form	06
OR			

Q. 10	a	Verify the following results on the matrix $\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$ (i) The sum of the eigenvalues of a matrix is the sum of the elements of the principal diagonal and (ii) The product of the eigenvalues of a matrix is equal to its determinant	07
	b	State Cayley – Hamilton theorem and use it to find A^{-1} and A^{-2} , if $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$	07
	c	Reduce the quadratic form $2xy + 2yz + 2zx$ into canonical form	06

Table showing the Bloom's Taxonomy Level, Course Outcome and Program Outcome				
Question		Bloom's Taxonomy Level attached	Course Outcome	Program Outcome
Q.1	(a)	L1	CO 01	
	(b)	L2	CO 01	
	(c)	L3	CO 01	
Q.2	(a)	L1	CO 01	
	(b)	L2	CO 01	
	(c)	L3	CO 01	
Q.3	(a)	L2	CO 02	
	(b)	L2	CO 02	
	(c)	L3	CO 02	
Q.4	(a)	L2	CO 02	
	(b)	L2	CO 02	
	(c)	L3	CO 02	
Q.5	(a)	L2	CO 03	
	(b)	L3	CO 03	
	(c)	L2	CO 03	

Q.6	(a)	L2	CO 03	
	(b)	L3	CO 03	
	(c)	L2	CO 03	
Q.7	(a)	L2	CO 04	
	(b)	L2	CO 04	
	(c)	L2	CO 04	
Q.8	(a)	L2	CO 04	
	(b)	L2	CO 04	
	(c)	L2	CO 04	
Q.9	(a)	L2	CO 06	
	(b)	L3	CO 05	
	(c)	L3	CO 05	
Q.10	(a)	L2	CO 06	
	(b)	L3	CO 05	
	(c)	L3	CO 05	
Lower order thinking skills				
Bloom's Taxonom y Levels	Remembering(knowledge):L ₁		Understanding Comprehension): L ₂	Applying (Application): L ₃
	Higher order thinking skills			
	Analyzing (Analysis):L ₄		Valuating (Evaluation): L ₅	Creating (Synthesis): L ₆