Model Question Paper-I with effect from 2021

USN | | | | | | | | | |

Second Semester B.E Degree Examination

Advanced Calculus and Numerical Methods (21MAT21)

TIME: 03 Hours **Max. Marks: 100**

Note: Answer any **FIVE** full questions, choosing at least **ONE** question from each module.

		Module -1	**Marks**
Q.01	a	Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)dxdydz$	**06**
	b	Evaluate $\int_{0}^{a}\int_{y}^{a}\frac{x}{x^2+y^2}dxdy$ by changing the order of integration	**07**
	c	Derive the relation between Gamma and Beta functions	**07**
		OR	
Q.02	a	Evaluate $\int_{0}^{1}\int_{0}^{\sqrt{1-y^2}}(x^2+y^2)dxdy$ by changing into polar coordinates	**06**
	b	Using double integration find the area between the parabolas $y^2=4ax$ and $x^2=4ay$	**07**
	c	Using beta and gamma functions, evaluate $\int_{0}^{\frac{\pi}{2}}\sqrt{\tan\theta}\, d\theta$	**07**
		Module-2	
Q. 03	a	Find the directional derivative of $\emptyset = x^2yz+4xz^2$ at the point $(1,-2,1)$ in the direction of the vector $2\hat{\imath}-\hat{\jmath}-2\hat{k}$	**06**
	b	Find $div\,\vec{F}$ and $curl\,\vec{F}$, where $\vec{F}=grad(x^3+y^3+z^3-3xyz)$	**07**
	c	Define an irrotational vector. Find the constants a, b and c such that $\vec{F}=(axy-z^3)\hat{\imath}+(bx^2+z)\hat{\jmath}+(bxz^2+cy)\hat{k}$ is irrotational.	**07**
		OR	
Q.04	a	Find the work done in moving a particle in the force field $\vec{F}=3x^2\hat{\imath}+(2xz-y)\hat{\jmath}+z\hat{k}$ along the straight line from $(0,0,0)$ to $(2,1,3)$.	**6**
	b	Apply Green's theorem to evaluate $\int_C [(3x-8y^2)dx+(4y-6xy)dy]$, where C is the boundary of the region bounded by $x=0, y=0, x+y=1$	**07**
	c	Apply Stoke's theorem to evaluate $\iint curl\,\vec{F}\cdot\hat{n}ds$, where $\vec{F}=(x^2+y^2)\hat{\imath}-2xy\hat{\jmath}$, taken around the rectangle bounded by the lines $x=\pm a, y=0, y=b$	**07**
		Module-3	
Q. 05	a	Form the partial differential equation by eliminating the arbitrary function from the relation $ax+by+cz=f(x^2+y^2+z^2)$.	**06**
	b	Solve $\frac{\partial^2 z}{\partial x^2}=xy$ subject to the conditions $\frac{\partial z}{\partial x}=\log(1+y)$, when $x=1$ and $z=0$, when $x=0$.	**07**
	c	With usual notations derive a one-dimensional heat equation	**07**
		OR	

<table>
<tr><td>Q. 06</td><td>a</td><td>Form the partial differential equation by eliminating the arbitrary constants from $(x-a)^2+(y-b)^2\ +z^2=4$</td><td>06</td></tr>
<tr><td></td><td>b</td><td>Solve $x^2(y-z)p+y^2(z-x)q=z^2(x-y)$</td><td>07</td></tr>
<tr><td></td><td>c</td><td>Solve $\frac{\partial^2 z}{\partial y^2}=z$, given that when $y=0, z=e^x and\ \frac{\partial z}{\partial y}=e^{-x}$</td><td>07</td></tr>
<tr><td colspan="3" align="center">Module-4</td><td></td></tr>
<tr><td>Q. 07</td><td>a</td><td>Find the root of the equation $xe^x=cosx$ which lies in the interval $(0,1)$ by Regula-Falsi method correct to four decimal places</td><td>06</td></tr>
<tr><td></td><td>b</td><td>Using Newton's backward interpolation formula find the value of y when $x=\ 6$ from the given table
<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>y</td><td>1</td><td>-1</td><td>1</td><td>-1</td><td>1</td></tr></table></td><td>07</td></tr>
<tr><td></td><td>c</td><td>Evaluate $\int_0^5 \frac{dx}{4x+5}$ by Simpson's (1/3)rd rule, dividing the interval into 10 equal parts</td><td>07</td></tr>
<tr><td colspan="3" align="center">OR</td><td></td></tr>
<tr><td>Q. 08</td><td>a</td><td>By Newton's-Raphson method find the root of $x\sin x+\cos x=0$ which is near to x = π</td><td>06</td></tr>
<tr><td></td><td>b</td><td>Using Lagrange's interpolation formula, fit a polynomial which passes through the points $(-1,0),(1,\ 2),(2,\ 9)$ and $(3,\ 8)$ and hence estimate the value of y when $x=\ 2.2$</td><td>07</td></tr>
<tr><td></td><td>c</td><td>Evaluate $\int_4^{5.2} \log x\, dx$ using Simpson's (3/8)th rule by taking 7 ordinates.</td><td>07</td></tr>
<tr><td colspan="3" align="center">Module-5</td><td></td></tr>
<tr><td>Q. 09</td><td>a</td><td>Using the Taylor series method find the approximate value of $y(0.1)$, from
$\frac{dy}{dx}=3x+y^2$, with $y(0)=1$</td><td>06</td></tr>
<tr><td></td><td>b</td><td>Apply the Runge-Kutta method to find $y(0.1)$, if
$\frac{dy}{dx}=\frac{y^2-x^2}{y^2+x^2}$, with $y(0)=1$</td><td>07</td></tr>
<tr><td></td><td>c</td><td>Using Milne's Predictor-Corrector method, find $y(4.5)$, given $\frac{dy}{dx}=\frac{2-y^2}{5x}$ and
$y(4.1)=1.0049$, $y(4.2)=1.0097$,
$y(4.3)=1.0143$, $y(4.4)=1.0187$</td><td>07</td></tr>
<tr><td colspan="3" align="center">OR</td><td></td></tr>
<tr><td>Q. 10</td><td>a</td><td>Using Modified Euler's method find y(0.1), taking h = 0.05, given that
$\frac{dy}{dx}=x^2+y, with\ y(0)=1.$</td><td>06</td></tr>
<tr><td></td><td>b</td><td>Using the Runge-Kutta method of order 4, find y(0.2), given that $\frac{dy}{dx}=3x+\frac{y}{2}$, y(0) = 1</td><td>07</td></tr>
<tr><td></td><td>c</td><td>Given $\frac{dy}{dx}=\frac{(1+x^2)y^2}{2}$, $y(0)=1, y(0.1)=1.06,\ y(0.2)=1.12,\ y(0.3)=1.21$, evaluate $y(0.4)$ by using Milne's predictor-corrector method.</td><td>07</td></tr>
</table>

Table showing the Bloom's Taxonomy Level, Course Outcome and Program Outcome				
Question		**Bloom's Taxonomy Level attached**	**Course Outcome**	**Program Outcome**
Q.1	(a)	L2	CO 01	PO 01
	(b)	L3	CO 01	PO 02
	(c)	L2	CO 01	PO 01
Q.2	(a)	L2	CO 01	PO 01
	(b)	L3	CO 01	PO 02
	(c)	L2	CO 01	PO 01
Q.3	(a)	L2	CO 02	PO 01
	(b)	L2	CO 02	PO 01
	(c)	L3	CO 02	PO 03
Q.4	(a)	L2	CO 02	PO 01
	(b)	L3	CO 02	PO 03
	(c)	L3	CO 02	PO 03
Q.5	(a)	L2	CO 03	PO 02
	(b)	L2	CO 03	PO 02
	(c)	L3	CO 03	PO 03
Q.6	(a)	L2	CO 03	PO 02
	(b)	L2	CO 03	PO 02
	(c)	L2	CO 03	PO 02
Q.7	(a)	L2	CO 04	PO 01
	(b)	L2	CO 04	PO 01
	(c)	L2	CO 04	PO 02
Q.8	(a)	L2	CO 04	PO 01

	(b)	L2	CO 04	PO 01
	(c)	L2	CO 04	PO 02
Q.9	(a)	L2	CO 05	PO 01
	(b)	L2	CO 05	PO 01
	(c)	L2	CO 05	PO 02
Q.10	(a)	L2	CO 05	PO 01
	(b)	L2	CO 05	PO 02
	(c)	L2	CO 05	PO 02

Bloom's Taxonomy Levels	**Lower order thinking skills**		
	Remembering (knowledge): L_1	Understanding (Comprehension): L_2	Applying (Application): L_3
	Higher order thinking skills		
	Analyzing (Analysis): L_4	Valuating (Evaluation): L_5	Creating (Synthesis): L_6