

Model Question Paper-II with effect from 2021

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Second Semester B.E Degree Examination

Advanced Calculus and Numerical Methods (21MAT21)

TIME: 03 Hours

Max. Marks: 100

Note: Answer any **FIVE** full questions, choosing at least **ONE** question from each module.

Module -1			Marks
Q.01	a	Evaluate $\int \int xy \, dx dy$ over the region bounded by the x-axis, ordinate $x = 2a$ and the curve $x^2 = 4ay$	06
	b	Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$, by using double integration	07
	c	Show that $\int_0^{\frac{\pi}{2}} \sqrt{\sin \theta} \, d\theta \times \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{\sin \theta}} = \pi$	07
OR			
Q.02	a	Change the order of integration and hence evaluate $\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y} \, dy dx$	06
	b	Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz \, dx dy dz$	07
	c	Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$	07
Module-2			
Q. 03	a	Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$	06
	b	If $\vec{F} = x^2y \hat{i} + y^2z \hat{j} + z^2x \hat{k}$, find $\text{Curl}(\text{Curl} \vec{F})$	07
	c	Show that $\vec{F} = (y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$ is both solenoidal and irrotational.	07
OR			
Q.04	a	If $\vec{F} = (5xy - 6x^2)\hat{i} + (2y - 4x)\hat{j}$, evaluate $\int_C \vec{F} \cdot d\vec{r}$ along the curve $C: y = x^3$ in the xy -plane from the point $(1, 1)$ to $(2, 8)$	06
	b	Using Green's theorem, evaluate $\int_C (xy + y^2) \, dx + x^2 \, dy$, where C is bounded by $y = x$ and $y = x^2$	07
	c	Using Stoke's theorem, evaluate $\oint_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = (x^2 + y^2) - 2xy$, taken around the rectangle whose vertices are $(a, 0)$, (a, b) , $(-a, b)$, $(-a, 0)$.	07
Module-3			
Q. 05	a	Form the partial differential equation from the relation $z = f(x + at) + g(x - at)$.	06
	b	Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial z}{\partial x} - 4z = 0$, given that when $x = 0$, $z = 1$ and $\frac{\partial z}{\partial x} = y$	07

	c	Derive one-dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$	07												
OR															
Q. 06	a	Form the partial differential equation from $f(x + y + z, x^2 + y^2 + z^2) = 0$	06												
	b	Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ for which $\frac{\partial z}{\partial y} = -2 \sin y$ when $x = 0$ and $z = 0$ when y is an odd multiple of $\frac{\pi}{2}$.	07												
	c	Solve $x(y^2 - z^2)p + y(z^2 - x^2)q - z(x^2 - y^2)r = 0$	07												
Module-4															
Q. 07	a	Find a real root of $x^3 - 9x + 1 = 0$ in $(2, 3)$ by the Regula-Falsi method in four iterations	06												
	b	Using Newton's forward interpolation find y at $x = 5$ from the data <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td> <td>4</td> <td>6</td> <td>8</td> <td>10</td> </tr> <tr> <td>y</td> <td>1</td> <td>3</td> <td>8</td> <td>16</td> </tr> </table>	x	4	6	8	10	y	1	3	8	16	07		
x	4	6	8	10											
y	1	3	8	16											
	c	Evaluate $\int_0^{\pi} \sqrt{\cos \theta} d\theta$ by taking 7 ordinates by Simpson's $(1/3)^{rd}$ rule.	07												
OR															
Q. 08	a	Using the Newton-Raphson method, find the root of $3x = \cos x + 1$ correct four decimal places.	06												
	b	Using Newton's divided difference interpolation find $f(9)$, Given that <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td> <td>5</td> <td>7</td> <td>11</td> <td>13</td> <td>17</td> </tr> <tr> <td>y</td> <td>150</td> <td>392</td> <td>1452</td> <td>2366</td> <td>5202</td> </tr> </table>	x	5	7	11	13	17	y	150	392	1452	2366	5202	07
x	5	7	11	13	17										
y	150	392	1452	2366	5202										
	c	Evaluate $\int_0^1 \frac{dx}{1+x^2}$, using Simpson's $(3/8)^{th}$ rule by taking 7 ordinates.	07												
Module-5															
Q. 09	a	Use the Taylor series method to find $y(0.2)$ from $\frac{dy}{dx} = x^2 y - 1$, with $y(0) = 1$	06												
	b	By using modified Euler's method, find $y(0.2)$, taking $h = 0.1$ from $\frac{dy}{dx} = \frac{y-x}{y+x}$, with $y(0) = 1$	07												
	c	Applying Milne's Predictor-Corrector method, find $y(0.8)$, from $\frac{dy}{dx} = x^3 + y$, given that $y(0) = 2$, $y(0.2) = 2.073$, $y(0.4) = 2.452$, $y(0.6) = 3.023$	07												
Q.10	a	Employ Taylor's series method to evaluate $y(0.2)$, taking $h = 0.1$ from $\frac{dy}{dx} = e^x - y^2$, with $y(0) = 1$	06												
	b	Using the Runge-Kutta method of order 4, find y at $x = 0.1$, given that $\frac{dy}{dx} = 3e^x + 2y$, $y(0) = 1$	07												
	c	Applying Milne's Predictor - Corrector method, to find $y(1.4)$, from $\frac{dy}{dx} = x^2 + \frac{y}{2}$, given that $y(1) = 2$, $y(1.1) = 2.2156$, $y(1.2) = 2.4549$, $y(1.3) = 2.7514$	07												

Table showing the Bloom's Taxonomy Level, Course Outcome and Program Outcome				
Question		Bloom's Taxonomy Level attached	Course Outcome	Program Outcome
Q.1	(a)	L2	CO 01	PO 02
	(b)	L3	CO 01	PO 03
	(c)	L2	CO 01	PO 02
Q.2	(a)	L2	CO 01	PO 02
	(b)	L2	CO 01	PO 02
	(c)	L2	CO 01	PO 02
Q.3	(a)	L2	CO 02	PO 01
	(b)	L2	CO 02	PO 02
	(c)	L2	CO 02	PO 02
Q.4	(a)	L2	CO 02	PO 02
	(b)	L3	CO 02	PO 03
	(c)	L3	CO 02	PO 03
Q.5	(a)	L2	CO 03	PO 01
	(b)	L2	CO 03	PO 02
	(c)	L3	CO 03	PO 03
Q.6	(a)	L2	CO 03	PO 02
	(b)	L2	CO 03	PO 02
	(c)	L2	CO 03	PO 02
Q.7	(a)	L2	CO 04	PO 01
	(b)	L2	CO 04	PO 01
	(c)	L2	CO 04	PO 02
Q.8	(a)	L2	CO 04	PO 01

	(b)	L2	CO 04	PO 01
	(c)	L2	CO 04	PO 02
Q.9	(a)	L2	CO 05	PO 01
	(b)	L2	CO 05	PO 01
	(c)	L2	CO 05	PO 02
Q.10	(a)	L2	CO 05	PO 01
	(b)	L2	CO 05	PO 02
	(c)	L2	CO 05	PO 02
Lower order thinking skills				
Bloom's Taxonomy Levels	Remembering (knowledge): L ₁	Understanding (Comprehension): L ₂	Applying (Application): L ₃	
	Higher-order thinking skills			
	Analyzing (Analysis): L ₄	Valuating (Evaluation): L ₅	Creating (Synthesis): L ₆	