Model Question Paper-II with effect from 2022

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Third Semester B.E Degree Examination
Transform Calculus, Fourier Series and Numerical Techniques
(21MAT31)

TIME: 03 Hours **Max. Marks: 100**

Note: Answer any **FIVE** full questions, choosing at least **ONE** question from each module.

		Module -1	**Marks**
Q.01	a	Find the Laplace transform of (i) $e^{-3t} sin5t\ cos3t$ (ii) $\frac{1-e^t}{t}$	**06**
	b	Find the Laplace transform of the square–wave function of period a given by $f(t) = \begin{cases} 1, & 0 < t < a/2 \\ -1, & a/2 < t < 2 \end{cases}$	**07**
	c	Using the convolution theorem find the inverse Laplace transform of $\frac{1}{(s^2+1)(s^2+9)}$	**07**
		OR	
Q.02	a	Using the unit step function, find the Laplace transform of $f(t) = \begin{cases} \cos t, & 0 \le t \le \pi \\ \cos 2t, & \pi \le t \le 2\pi \\ \cos 3t, & t \ge 2\pi \end{cases}$	**06**
	b	Find the inverse Laplace transform of $\frac{2s^2-6s+5}{s^3-6s^2+11s-6}$	**07**
	c	Solve by using Laplace transform techniques $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t$ with $x(0) = 2\ and\ x'(0) = -1$	**07**
		Module-2	
Q. 03	a	Find a Fourier series to represent $f(x) = x^2 \quad in \quad -\pi \le x \le \pi$	**06**
	b	Obtain the half-range cosine series for $f(x) = x\ sinx$ in $(0,\ \pi)$ and hence show that $\frac{\pi - 2}{4} = \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \cdots \infty$	**07**
	c	The following table gives the variation of periodic current over a period. *t sec*: 0, T/6, T/3, T/2, 2T/3, 5T/6, T *A amp*: 1.98, 1.30, 1.05, 1.30, -0.88, -0.25, 1.98 Show that there is a direct current part of 0.75 amp in the variable current and obtain the amplitude of the first harmonic.	**07**

<table>
<tr><td colspan="3">OR</td><td></td></tr>
<tr><td>Q.04</td><td>a</td><td>Find the Fourier series expansion of $f(x) = 2x - x^2$, $in\ (0,\ 3)$</td><td>6</td></tr>
<tr><td></td><td>b</td><td>Obtain half-range sine series for
$f(x) = \begin{cases} kx\ , & 0 \le x \le \frac{l}{2} \\ k(l-x)\ , & \frac{l}{2} \le x \le l \end{cases}$</td><td>07</td></tr>
<tr><td></td><td>c</td><td>Expand y as a Fourier series up to the first harmonic if the values of y are given by
<table>
<tr><td>x</td><td>0°</td><td>30°</td><td>60°</td><td>90°</td><td>120°</td><td>150°</td><td>180°</td><td>210°</td><td>240</td><td>270</td><td>300</td><td>330</td></tr>
<tr><td>y</td><td>1.80</td><td>1.10</td><td>0.30</td><td>0.16</td><td>1.50</td><td>1.30</td><td>2.16</td><td>1.25</td><td>1.30</td><td>1.52</td><td>1.76</td><td>2.00</td></tr>
</table></td><td>07</td></tr>
<tr><td colspan="3">Module-3</td><td></td></tr>
<tr><td>Q. 05</td><td>a</td><td>Find the Fourier transform of $f(x) = \begin{cases} 1\ , & |x| \le 1 \\ 0\ , & |x| > 1 \end{cases}$
Hence evaluate $\int_0^\infty \frac{sinx}{x} dx$</td><td>06</td></tr>
<tr><td></td><td>b</td><td>Find the Fourier cosine and sine transforms of e^{-ax}</td><td>07</td></tr>
<tr><td></td><td>c</td><td>Find the Z-transforms of $(i)\ (n+1)^2$ and $(ii)\ \sin(3n+5)$</td><td>07</td></tr>
<tr><td colspan="3">OR</td><td></td></tr>
<tr><td>Q. 06</td><td>a</td><td>Find the Fourier transform of $e^{-a^2x^2}, a > 0$. Hence deduce that it is self-reciprocal in respect of Fourier series</td><td>06</td></tr>
<tr><td></td><td>b</td><td>Find the inverse z –transform of $\frac{2z^2+3z}{(z+2)(z-4)}$</td><td>07</td></tr>
<tr><td></td><td>c</td><td>Using z-transformation, solve the difference equation $u_{n+2} + 4u_{n+1} + 3u_n = 3^n$,
$u_0 = 0\ , u_1 = 1$</td><td>07</td></tr>
<tr><td colspan="3">Module-4</td><td></td></tr>
<tr><td>Q. 07</td><td>a</td><td>Classify the following partial differential equations
(i) $u_{xx} + 4u_{xy} + 4u_{yy} - u_x + 2u_y = 0$
(ii) $x^2u_{xx} + (1-y^2)u_{yy} = 0\ , -1 < y < 1$
(iii) $(1+x^2)u_{xx} + (5+2x^2)u_{xt} + (4+x^2)u_{tt} = 0$
(iv) $y^2u_{xx} - 2yu_{xy} + u_{yy} - u_y = 8y$</td><td>10</td></tr>
<tr><td></td><td>b</td><td>Find the values of $u(x,t)$ satisfying the parabolic equation $u_t = 4u_{xx}$ and the boundary conditions $u(0,t) = 0 = u(8,0)$ and $u(x,0) = 4x - \frac{x^2}{2}$ at the points
$x = i :\ i = 0,1,2, \ldots ,8$ and $t = \frac{j}{8} : j = 0,1,2,3,4.$</td><td>10</td></tr>
<tr><td colspan="3">OR</td><td></td></tr>
<tr><td>Q. 08</td><td>a</td><td>Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u(x,0) = \sin \pi x\ , 0 \le x \le 1$
$u(0,t) = u(1,t) = 0$, Carry out computations for two levels, taking
$h = \frac{1}{34} and\ k = \frac{1}{36}$</td><td>10</td></tr>
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	b	The transverse displacement u of a point at a distance x from one end and at any time t of a vibrating string satisfies the equation $u_{tt} = 25\, u_{xx},$ with the boundary conditions $u(x,t) = u(5,t) = 0$ and the initial conditions $u(x,0) = \begin{cases} 20x\,, & 0 \le x \le 1 \\ 5(5-x), & 1 \le x \le 5 \end{cases}$ and $u_t(x,0) = 0$. Solve this equation numerically up to $t = 5$ taking $h = 1, k = 0.2$.	**10**
		Module-5	
Q. 09	a	Using Runge –Kutta method of order four, solve $\frac{d^2y}{dx^2} = y + x\frac{dy}{dx}$ for x = 0.2 , Given that , $y(0) = 1\ \ , y'(0) = 0$	**06**
	b	Find the extremals of the functional $\int_{x_1}^{x_2}[y^2 + (y')^2 + 2ye^x]dx$	**07**
	c	Find the path on which a particle in the absence of friction, will slide from one point to another in the shortest time under the action of gravity	**07**
		OR	
Q. 10	a	Apply Milne's method to solve $\frac{d^2y}{dx^2} = 1 + \frac{dy}{dx}$ at x = 0.4. given that $y(0) = 1$, $y(0.1) = 1.1103$, $y(0.2) = 1.2427$, $y(0.3) = 1.399$ $y'(0) = 1\ \ , y'(0.1) = 1.2103, y'(0.2) = 1.4427, y'(0.3) = 1.699$	06
	b	Find the extremals of the functional $\int_{x_1}^{x_2}\frac{(y')^2}{x^3}dx$	**07**
	c	Find the curve on which the functional $\int_0^{\pi/2}[(y')^2 + 12xy]\,dx$ with $y(0) = 0\ and\ y(\pi/2) = 0$ can be extremised	**07**

Table showing the Bloom's Taxonomy Level, Course Outcome and Program Outcome

Question		Bloom's Taxonomy Level attached	Course Outcome	Program Outcome
Q.1	(a)	L1	CO 01	PO 01
	(b)	L2	CO 01	PO 02
	(c)	L2	CO 01	PO 02
Q.2	(a)	L2	CO 01	PO 02
	(b)	L2	CO 01	PO 02
	(c)	L2	CO 01	PO 02
Q.3	(a)	L2	CO 02	PO 02
	(b)	L2	CO 02	PO 02
	(c)	L3	CO 02	PO 02
Q.4	(a)	L2	CO 02	PO 02
	(b)	L2	CO 02	PO 02
	(c)	L2	CO 02	PO 02
Q.5	(a)	L2	CO 03	PO 02
	(b)	L2	CO 03	PO 02
	(c)	L2	CO 03	PO 02
Q.6	(a)	L2	CO 03	PO 02

	(b)	L2	CO 03	PO 02
	(c)	L3	CO 03	PO 02
Q.7	(a)	L1	CO 04	PO 01
	(b)	L2	CO 04	PO 02
Q.8	(a)	L2	CO 04	PO 02
	(b)	L3	CO 04	PO 02
Q.9	(a)	L2	CO 05	PO 01
	(b)	L2	CO 05	PO 02
	(c)	L3	CO 05	PO 02
Q.10	(a)	L2	CO 05	PO 01
	(b)	L2	CO 05	PO 02
	(c)	L2	CO 05	PO 02

Bloom's Taxonomy Levels	**Lower order thinking skills**		
	Remembering (knowledge): L_1	Understanding (Comprehension): L_2	Applying (Application): L_3
	Higher-order thinking skills		
	Analyzing (Analysis): L_4	Valuating (Evaluation): L_5	Creating (Synthesis): L_6