Model Question Paper-I with effect from 2022

USN | | | | | | | | | | |

Fourth Semester B.E Degree Examination
Mathematical Foundations for Computing, Probability & Statistics
Computer Science & Allied Engg. branches-21MATCS41

TIME: 03 Hours **Max. Marks: 100**

Note: Answer any **FIVE** full questions, choosing at least **ONE** question from each module.

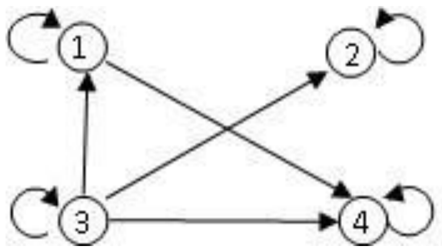
Q.No.		Question	M	L	CO
		Module -1			
01	a	Define tautology. Determine whether the following compound statement is a tautology or not. $\{(p \vee q) \rightarrow r\} \leftrightarrow \{\neg r \rightarrow \neg(p \vee q)\}$	**06**	**L2**	**CO1**
	b	Using the laws of logic, prove the following logical equivalence $[(\neg p \vee \neg q) \wedge (F_0 \vee p) \wedge p] \Leftrightarrow p \wedge \neg q$.	**07**	**L3**	**CO1**
	c	Give direct proof and proof by contradiction for the statement "If n is an odd integer then $n+9$ is an even integer"	**07**	**L2**	**CO1**
		OR			
02	a	Test the validity of the arguments using rules of inference. $(\neg p \vee q) \rightarrow r$ $r \rightarrow (s \vee t)$ $\neg s \wedge \neg u$ $\underline{\neg u \rightarrow \neg t}$ $\therefore p$	**06**	**L3**	**CO1**
	b	Find whether the following arguments are valid or not for which the universe is the set of all triangles. In triangle XYZ, there is no pair of angles of equal measure. If the triangle has two sides of equal length, then it is isosceles. If the triangle is isosceles, then it has two angles of equal measure. Therefore Triangle XYZ has no two sides of equal length.	**07**	**L3**	**CO1**
	c	If $p(x): x \geq 0, q(x): x^2 \geq 0, r(x): x^2 - 3x - 4 = 0, s(x): x^2 - 3 > 0$ Determine the truth or falsity of the following statement: $i)\ \exists x[p(x) \wedge q(x)]$ $ii)\ \forall x[p(x) \rightarrow q(x)]$ $iii)\ \forall x[q(x) \rightarrow s(x)]$ $iv)\ \forall x[r(x) \wedge s(x)]$ $v)\ \exists x[p(x) \wedge r(x)]$ $vi)\ \forall x[r(x) \rightarrow p(x)]$ $vii)\ \exists x[r(x) \rightarrow \neg p(x)]$	**07**	**L2**	**CO1**
		Module-2			
03	a	Let f and g be functions from R to R defined by $f(x) = ax + b$ and $g(x) = 1 - x + x^2$, If $(g \circ f)(x) = 9x^2 - 9x + 3$ determine a and b.	**06**	**L2**	**CO2**

b Let $A=\{1,2,3,4,6\}$ and R be a relation on A defined by aRb if and only if " a is a multiple of b". Write down the relation R, relation matrix $M(R)$ and draw its digraph. — **07** | **L2** | **CO2**

c Prove that in every graph the number of vertices of odd degree is even. — 07 | **L2** | **CO2**

OR

4 a The digraph of a relation R defined on the set $A=\{1,2,3,4\}$ is shown below. Verify that (A, R) is a poset and construct the corresponding Hasse diagram. — **06** | **L2** | **CO2**

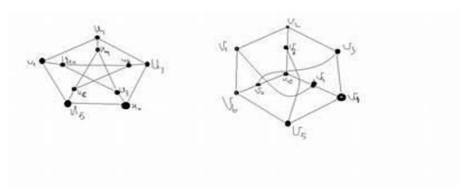


b Let $A = B = C = R$, and $f : A \to B$ *and* $g : B \to C$ be defined by

$$f(a) = 2a+1,\ g(b) = \frac{1}{3}b, \forall a \in A, \forall b \in B.$$

Compute $g \circ f$ and show that $g \circ f$ is invertible. What is $(g \circ f)^{-1}$? — **07** | **L2** | **CO2**

c Define Graph isomorphism. Determine whether the following graphs are isomorphic or not. — **07** | **L2** | **CO2**



Module-3

5 a Ten competitors in a beauty contest are ranked by two judges A and B in the following order: — **06** | **L2** | **CO3**

ID No. of competitors	1	2	3	4	5	6	7	8	9	10
Judge A	1	6	5	10	3	2	4	9	7	8
Judge B	6	4	9	8	1	2	3	10	5	7

Calculate the rank correlation coefficient.

b In a partially destroyed laboratory record, the lines of regression of *y* on *x* and *x* on *y* are available as $4x - 5y + 33 = 0$ *and* $20x - 9y = 107$. Calculate $\bar{x}$ *and* $\bar{y}$ and the coefficient of correlation between *x* and *y*. — **07** | **L2** | **CO3**

c An experiment gave the following values: — **07** | **L2** | **CO3**

v(ft/min)	350	400	500	600
t(min.)	61	26	7	26

It is known that v and t are connected by the relation $v = at^b$. Find the best possible values of *a* and *b*.

OR					
6	a	The following table gives the heights of fathers(x) and sons (y): x: 65, 66, 67, 67, 68, 69, 70, 72 y: 67, 68, 65, 68, 72, 72, 69, 71 Find the lines of regression and Calculate the coefficient of correlation.	**06**	**L2**	**CO3**
	b	Fit a parabola $y = ax^2 + bx + c$ for the data x: 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0 y: 1.1, 1.3, 1.6, 2.0, 2.7, 3.4, 4.1	**07**	**L2**	**CO3**
	c	With usual notation, compute means $\bar{x}, \bar{y}$ and correlation coefficient r from the following lines of regression: $2x+3y+1=0$ *and* $x+6y-4=0$.	**07**	**L2**	**CO3**
Module-4					
7	a	A random variable X has the following probability function: x: -2, -1, 0, 1, 2, 3 $P(x)$: 0.1, k, 0.2, 2k, 0.3, k Find the value of k and calculate the mean and variance	**06**	**L2**	**CO4**
	b	Find the mean and standard deviation of the Binomial distribution	**07**	**L2**	**CO4**
	c	In a test on 2000 electric bulbs, it was found that the life of a particular make was normally distributed with an average life of 2040 hours and Standard deviation of 60 hours. Estimate the number of bulbs likely to burn for i. More than 2150 hours ii. Less than 1950 hours iii. Between 1920 and 2160 hours	**07**	**L3**	**CO4**
OR					
8	a	Find the constant k such that $f(x) = \begin{cases} kx^2 & 0 < x < 3 \\ 0 & otherwise \end{cases}$ is a p.d.f. Also, compute i) $P(1 < x < 2)$ ii) $P(x \le 1)$ iii) $P(x > 1)$	**06**	**L2**	**CO4**
	b	2% of fuses manufactured by a firm are found to be defective. Find the probability that a box containing 200 fuses contains (i) no defective fuses (ii) 3 or more defective fuses (iii) at least one defective fuse.	**07**	**L2**	**CO4**
	c	In a normal distribution 31% of the items are under 45 and 8% of the items are over 64. Find the mean and S.D of the distribution.	**07**	**L2**	**CO4**
Module-5					
9	a	The joint distribution of two random variables X and Y is as follows X \ Y: -4, 2, 7 1: 1/8, 1/4, 1/8 5: 1/4, 1/8, 1/8 Compute the following. (i) E(X) and E(Y) (ii) E(XY) (iii) σ_X and σ_Y (iv) COV(X,Y) (v) $\rho(X,Y)$	**06**	L2	**CO5**

<table>
<tr><td></td><td>b</td><td>A coin was tossed 400 times and head turned up 216 times. Test the hypothesis that the coin is unbiased at 5% level of significance.</td><td>07</td><td>L2</td><td>CO5</td></tr>
<tr><td></td><td>c</td><td>A certain stimulus administered to each of the 12 patients resulted in the following change in blood pressure 5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, and 4. Can it be concluded that the stimulus will increase the blood pressure? ($t_{.05}$ for 11 d.f = 2.201)</td><td>07</td><td>L3</td><td>CO5</td></tr>
<tr><td colspan="5">OR</td><td></td></tr>
<tr><td>10</td><td>a</td><td>Explain the terms: (i) Null hypothesis (ii) Confidence intervals (iii) Type-I and Type-II errors.</td><td>06</td><td>L2</td><td>CO5</td></tr>
<tr><td></td><td>b</td><td>The mean life of 100 fluorescent tube lights manufactured by a company is found to be 1570 hrs with a standard deviation of 120 hrs. Test the hypothesis that the mean lifetime of the lights produced by the company is 1600 hrs at 0.01 level of significance.</td><td>07</td><td>L3</td><td>CO5</td></tr>
<tr><td></td><td>c</td><td>A die is thrown 264 times and the number appearing on the face(x) follows the following frequency distribution.
<table>
<tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>y</td><td>40</td><td>32</td><td>28</td><td>58</td><td>54</td><td>60</td></tr>
</table>
Calculate the value of χ^2.</td><td>07</td><td>L3</td><td>CO5</td></tr>
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<tr><td colspan="4"></td></tr>
<tr><td rowspan="4">Bloom's Taxonomy Levels</td><td colspan="3">Lower-order thinking skills</td></tr>
<tr><td>Remembering (knowledge): L_1</td><td>Understanding (Comprehension): L_2</td><td>Applying (Application): L_3</td></tr>
<tr><td colspan="3">Higher-order thinking skills</td></tr>
<tr><td>Analyzing (Analysis): L_4</td><td>Valuating (Evaluation): L_5</td><td>Creating (Synthesis): L_6</td></tr>
</table>

Model Question Paper-II with effect from 2022

USN | | | | | | | | | | |

Fourth Semester B.E Degree Examination
Mathematical Foundations for Computing, Probability & Statistics
(Computer Science & Allied Engg. branches)-21MATCS41

TIME: 03 Hours **Max. Marks: 100**

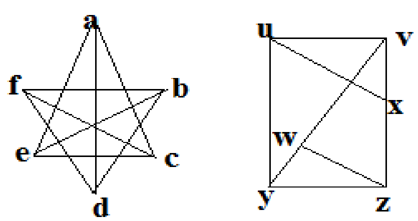
Note: Answer any **FIVE** full questions, choosing at least **ONE** question from each module.

Q.No.		Question	M	L	CO
		Module -1			
1	a	Define tautology. Show that $\{(p \vee q) \wedge (p \to r) \wedge (q \to r)\} \to r$ is a tautology by constructing the truth table.	06	L2	CO1
	b	Prove the following using the laws of logic $[\neg p \wedge (\neg q \wedge r)] \vee [(q \wedge r) \vee (p \wedge r)] \Leftrightarrow r$.	07	L3	CO1
	c	For any two odd integers m and n, show that i) m + n is even ii) mn is odd.	07	L2	CO1
		OR			
2	a	Define i) open statement ii) Quantifiers	06	L2	CO1
	b	Write the following argument in symbolic form and then establish the validity: If A gets the Supervisor's position and works hard, then he will get a raise. If he gets a raise, then he will buy a car. He has not purchased a car. Therefore he did not get the Supervisor's position or he did not work hard.	07	L3	CO1
	c	For the following statements, the universe comprises all non-zero integers. Determine the truth value of each statement. $a)\ \exists x \exists y[xy = 1]$ $b)\ \exists x \forall y[xy = 1]$ $c)\ \forall x \exists y[xy = 1]$ $d)\ \exists x \exists y[(2x + y = 5) \wedge (x - 3y = -8)]$ $e) \exists x \exists y[(3x - y = 7) \wedge (2x + 4y = 3)]$	07	L2	CO1
		Module-2			
3	a	Let $A = \{1, 2, 3, 4\}$ *and* $B = \{1, 2, 3, 4, 5, 6\}$ i) How many functions are there from A to B? How many of these are one-to-one? How many are onto? ii) How many functions are there from B to A? How many of these are onto? How many are one-to-one?	06	L2	CO2
	b	Let $A = \{1, 2, 3, 4, 5\} \times \{1, 2, 3, 4, 5\}$ and define R on A by $(x_1, y_1) R (x_2, y_2)$ *if* $x_1 + y_1 = x_2 + y_2$ i) Verify that R is an equivalence relation on A ii) Determine the equivalence classes [(1, 3)], [(2.4)] and [(1, 1)].	07	L3	CO2

	c	Define i) Simple graph ii) Complete graph iii) Sub graph iv) Spanning sub graph v) Induced subgraph vi) Complement of a graph vii) Euler Circuit. Give one example each.	**07**	**L2**	**CO2**

OR

4	a	Draw the Hasse diagram representing the positive divisors of 36.	**06**	**L2**	**CO2**
	b	Let $f : R \to R$ be defined by $f(x) = \begin{cases} 3x-5 & for\ x > 0 \\ 1-3x & for\ x \le 0 \end{cases}$. Find $f^{-1}(0), f^{-1}(1), f^{-1}(3), f^{-1}([-5,5])$.	**07**	**L2**	**CO2**
	c	Define Graph isomorphism. Determine whether the following graphs are isomorphic or not.	**07**	**L3**	**CO2**



Module-3

5	a	Calculate the coefficient of correlation and obtain the lines of regression for the following data:	**06**	**L2**	**CO3**

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15

	b	Fit a curve $y = ax^b$ for the following data.	**07**	**L2**	**CO3**

x	1	2	3	4	5
y	0.5	2	4.5	8	12.5

	c	Fit a straight line in the least square sense for the following data	**07**	**L2**	**CO3**

x	50	70	100	120
y	12	15	21	25

OR

6	a	The following are the percentage of marks in Mathematics(x) and Statistics (y) of nine students. Calculate the rank correlation coefficient.	**06**	**L2**	**CO3**

x	38	50	42	61	43	55	67	46	72
y	41	64	70	75	44	55	62	56	60

	b	Fit a second-degree parabola $y = ax^2 + bx + c$ for the data and hence estimate y at x = 6.	**07**	**L2**	**CO3**

x	1	2	3	4	5
y	10	12	13	16	19

	c	With usual notation, compute $\bar{x}, \bar{y}$ and coefficient of correlation r from the following lines of regression: $y = 0.516x + 33.73$ *and* $x = 0.512y + 32.52$.	**07**	**L2**	**CO3**

Module-4					
7	a	A random variable X has the following probability function: x: 0, 1, 2, 3, 5, 6, 7 $P(x)$: 0, k, 2k, 3k, k^2, $2k^2$, $7k^2$+ k Find k and evaluate: $P(X<6), P(X\geq 6)$ *and* $P(0<X<5)$.	**06**	**L3**	**CO4**
	b	Find the mean and standard deviation of Poisson distribution	**07**	**L2**	**CO4**
	c	The marks of 1000 students in an examination follow a normal distribution with a mean 70 and a standard deviation 5. Find the number of students whose marks will be (i) less than 65 (ii) more than 75 (iii)between 65 and 75.	**07**	**L2**	**CO4**
OR					
8	a	Find the constant k such that $f(x)=\begin{cases} kxe^{-x} & 0<x<1 \\ 0 & otherwise \end{cases}$ is a p.d.f. Find the mean.	**06**	**L2**	**CO4**
	b	The probability that a pen manufactured by a company will be defective is 1/10. If 12 such pens are manufactured, find the probability that (a) exactly two will be defective (b) at least two will be defective (c) none will be defective.	**07**	**L2**	**CO4**
	c	If the probability of a bad reaction from a certain injection is 0.001, determine the chance that out of 2000 individuals more than two will get a bad reaction.	**07**	**L2**	**CO4**
Module-5					
9	a	X and Y are independent random variables. X takes values 2, 5, and 7 with probability 1/2, 1/4, and 1/4 respectively. Y take values 3, 4, and 5 with the probability 1/3, 1/3 and 1/3. a) Find the joint probability distribution of X and Y. b) Show that the covariance of X and Y is equal to zero.	**06**	L2	**CO5**
	b	A coin was tossed 400 times and head turned up216 times. Test the hypothesis that the coin is unbiased at 5% level of significance.	**07**	L3	**CO5**
	c	In experiments on pea breeding, the following frequencies of seeds were obtained Round & Yellow: 315; Wrinkled & Yellow: 101; Round & green: 108; Wrinkled & green: 32; Total: 556 Theory predicts that the frequencies should be in proportions 9:3:3:1.Examine the correspondence between theory and experiment.	**07**	L3	**CO5**
OR					
10	a	Explain the terms (i) Null hypothesis (ii) Significance level (iii) Type I and Type II errors.	**06**	L2	**CO5**
	b	A sample of 100 students is taken from a large population. The mean height of the students in this sample is 160 cm. Can it be reasonably regarded that in the population the mean height is 165 cm and the standard deviation is 10 cm at 5% level of significance?	**07**	L3	**CO5**

	c	The nine items of a sample have the following values: 45, 47, 50, 52, 48, 47, 49, 53, 51. Does the mean of these differ significantly from the assumed mean of 47.5?($t_{0.05}$=2.31 for 8 degree of freedom)	**07**	L3	**CO5**

Bloom's Taxonomy Levels	**Lower-order thinking skills**		
	Remembering (knowledge): L_1	Understanding (Comprehension): L_2	Applying (Application): L_3
	Higher-order thinking skills		
	Analyzing (Analysis): L_4	Valuating (Evaluation): L_5	Creating (Synthesis): L_6