Model Question Paper-I with effect from 2022

USN | | | | | | | | | | |

Fourth Semester B.E Degree Examination
Complex Analysis, Probability & Linear Programming
(Mechanical Engg. Allied branches)-21MATME41

TIME: 03 Hours **Max. Marks: 100**

Note: Answer any **FIVE** full questions, choosing at least **ONE** question from each module.

Q.No.		Question	M	L	CO
		Module -1			
01	a	With usual notations, derive the Cauchy-Riemann equation in the Cartesian form	**06**	**L2**	**CO1**
	b	If f(z) is regular function of z, prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\lvert f(z)\rvert^2 = 4\lvert f'(z)\rvert^2$	**07**	**L2**	**CO1**
	c	Determine the analytical function whose real part is $y + e^x \cos y$	**07**	**L2**	**CO1**
		OR			
02	a	If $f(z)$ is an analytic function with constant modulus, show that $f(z)$ is constant.	**06**	**L2**	**CO1**
	b	Show that $w = \log z$ is analytic everywhere except at $z = 0$ and hence find its derivative.	**07**	**L2**	**CO1**
	c	Find the analytical function whose imaginary part is $e^{-x}(x\, siny - y \cos y)$	**07**	**L2**	**CO1**
		Module-2			
03	a	Discuss the transformation $w = e^z$	**06**	**L3**	**CO2**
	b	State and prove the Cauchy Integral theorem	**07**	**L2**	**CO2**
	c	Find the bilinear transformation which maps the points $z = 1, i, -1$ onto the points $\omega = i, 0, -i$	07	**L2**	**CO2**
		OR			
4	a	Find the bilinear transformation which maps $1,\ i, -1\ to\ 2,\ i, -2$ respectively.	**06**	**L2**	**CO2**
	b	Verify Cauchy's theorem for the integral of z^3 over the boundary of the rectangle with vertices $z = -1,\ 1,\ 1+i,\ -1+i$	**07**	**L2**	**CO2**
	c	Evaluate $\oint \frac{e^{-z}}{(z-1)(z-2)^2} dz$, over the curve $\lvert z\rvert = 3$	**07**	**L3**	**CO2**
		Module-3			
5	a	A random variable X has the following probability function: x: -2, -1, 0, 1, 2, 3 $P(x)$: 0.1, k, 0.2, 2k, 0.3, k Find the value of k and calculate the mean and variance	**06**	**L2**	**CO3**
	b	Find the mean and standard deviation of the Binomial distribution	**07**	**L2**	**CO3**
	c	In a certain factory turning out razor blades, there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 20. Use Poisson distribution to calculate the approximate number of packets containing no defective, one defective and two defective blades respectively in a consignment of 500 packets	**07**	**L3**	**CO3**

		OR			
6	a	The diameter of an electric cable is assumed to be a continuous variable with p.d.f $$f(x) = \begin{cases} 6x(1-x), & 0 \le x \le 1 \\ 0, & elsewhere \end{cases}$$ Verify that the above is a valid p.d.f. Also, find its mean and variance.	**06**	**L2**	**CO3**
	b	In a test on 2000 electric bulbs, it was found that the life of a particular make was normally distributed with an average life of 2040 hours and Standard deviation of 60 hours. Estimate the number of bulbs likely to burn for i. More than 2150 hours ii. Less than 1950 hours iii. Between 1920 and 2160 hours	**07**	**L3**	**CO3**
	c	The life of a T.V tube manufactured by a company is known to have a mean of 200 months. Assuming that the life has an exponential distribution, find the probability that the life of a tube manufactured by the company is i. Less than 200 months ii. Between 100 and 300 months iii. More than 200 months	**07**	**L3**	**CO3**
		Module-4			
7	a	Using Simplex method solve the L.P.P $Maximize\ Z = 3x_1 + 2x_2$, subject to: $2x_1 + x_2 \le 5$ $x_1 + x_2 \le 3$ $x_1, x_2 \ge 0$	**10**	**L3**	**CO4**
	b	Using Big –M method , solve the LPP $Minimize\ Z = 2x_1 + x_2$, subject to: $3x_1 + x_2 = 3$ $4x_1 + 3x_2 \ge 6$ $x_1 + 2x_2 \le 3$ $x_1, x_2 \ge 0$	**10**	**L3**	**CO4**
		OR			
8	a	Explain the canonical form and standard form of an LPP. Convert the following LPP to the standard form $Maximize\ Z = 3x_1 + 5x_2 + 7x_3$, subject to: $6x_1 - 4x_2 \le 5$ $3x_1 + 2x_2 + 5x_3 \ge 11$ $4x_1 + 3x_3 \le 2$ $x_1, x_2 \ge 0$ $x_1, x_2 \ge 0$	**10**	**L3**	**CO4**
	b	Use two –Phase method to solve the LPP $Maximize\ Z = 9x_1 + 3x_2$, subject to: $4x_1 + x_2 \le 8$ $2x_1 + x_2 \le 4$ $x_1, x_2 \ge 0$	**10**	**L3**	**CO4**
		Module-5			

Q.No		Question	Marks	BL	CO
9	a	Solve the following transportation problem	**10**	L3	**CO5**

Source		A	B	C	D	Availability
		Destination				
	I	21	16	25	13	**11**
	II	17	18	14	23	**13**
	III	33	27	18	41	**19**
	Requirements	**6**	**10**	**12**	**15**	**43**

Q.No		Question	Marks	BL	CO
	b	Solve the assignment problem	**10**	L3	**CO5**

		Machines			
		M_1	M_2	M_3	M_4
Jobs	J_1	2	3	4	5
	J_2	4	5	6	7
	J_3	7	8	9	8
	J_4	3	5	8	4

Assign the jobs to different machines so as to minimize the total cost

OR

Q.No		Question	Marks	BL	CO
10	a	Obtain an initial basic solution to the following transportation problem	**10**	L3	**CO5**

From		To				
		A	B	C	D	Availability
	I	11	13	17	14	250
	II	16	18	14	10	300
	III	21	24	13	10	400
	Requirements	**200**	**225**	**275**	**250**	

Q.No		Question	Marks	BL	CO
	b	Five men are available to do five different jobs. From past records, the time (in hours) that each man takes to do each job is given below	**10**	L3	**CO5**

		Jobs				
		I	II	III	IV	V
Man	A	2	9	2	7	1
	B	6	8	7	6	1
	C	4	6	5	3	1
	D	4	2	7	3	1
	E	5	3	9	5	1

Find the assignment of men to jobs that will minimize the total time taken

Bloom's Taxonomy Levels	**Lower-order thinking skills**		
	Remembering (knowledge): L_1	Understanding (Comprehension): L_2	Applying (Application): L_3
	Higher-order thinking skills		

	Analyzing (Analysis): L_4	Valuating (Evaluation): L_5	Creating (Synthesis): L_6

Model Question Paper-II with effect from 2022

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Fourth Semester B.E Degree Examination

Complex Analysis, Probability & Linear Programming

(Mechanical Engg. And Allied branches)-21MATME41

TIME: 03 Hours **Max. Marks: 100**

Note: Answer any **FIVE** full questions, choosing at least **ONE** question from each module.

Q.No.		Question	M	L	CO
		Module -1			
01	a	With usual notations, derive the Cauchy-Riemann equation in the polar form	**06**	**L2**	**CO1**
	b	Find the constants a, b, c and d if $f(z) = (ax^4 + bx^2y^2 + cy^4 + dx^2 - 2y^2) + i(4x^3y - exy^3) + 4xy)$ is analytic	**07**	**L2**	**CO1**
	c	Determine the analytical function whose real part is $e^{2x}(x\cos 2y - y\sin 2y)$	**07**	**L2**	**CO1**
		OR			
02	a	Show that z^n is analytic. Hence, find its derivative	**06**	**L2**	**CO1**
	b	If f(z) is analytic function show that $\left(\frac{\partial}{\partial x}\lvert f(z)\rvert\right)^2 + \left(\frac{\partial}{\partial y}\lvert f(z)\rvert\right)^2 = \lvert f'(z)\rvert^2$	**07**	**L2**	**CO1**
	c	Find the regular function whose imaginary part is $e^x siny$	**07**	**L2**	**CO1**
		Module-2			
03	a	Discuss the transformation $w = z^2$	**06**	**L3**	**CO2**
	b	State and prove Cauchy Integral formula	**07**	**L2**	**CO2**
	c	Find the bilinear transformation which maps the points $z = 1, i, -1$ onto the points $\omega = 0, 1, \infty$	07	**L2**	**CO2**
		OR			
4	a	Evaluate $\int_c \frac{z^2-z+1}{Z-1} dx$, where C is the circle (i) $\lvert z\rvert = 1$ (ii) $\lvert z\rvert = \frac{1}{2}$	**06**	**L3**	**CO2**
	b	Discuss the transformation $w = z + \frac{1}{z}$	**07**	**L3**	**CO2**
	c	Evaluate $\oint \frac{\sin \pi z + \cos \pi z}{(z-1)(z-2)(z-3)} dz$, over the curve $\lvert z\rvert = 4$	**07**	**L3**	**CO2**
		Module-3			
5	a	A random variable X has the following probability function: (see table below) Find the value of k and evaluate i. $P[0 < X < 5]$ ii. $P[X < 6]$ iii. $P[X > 2]$	**06**	**L2**	**CO3**
	b	Find the mean and variance of a Poisson distribution	**07**	**L2**	**CO3**

Table for Q5 a:

x	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

	c	If the probability that a new-born child is a male is 0.6, Using Binomial distribution find the probability in a family of 5 children i. There is no boy ii. There is at least one boy iii. There are exactly 3 boys	**07**	**L3**	**CO3**
OR					
6	a	The p.d.f of a continuous random variable X is given by $f(x) = \begin{cases} k\,x^3 & 0 \le x \le 1 \\ 0\,, & elsewhere \end{cases}$ Find (i) The value of k (ii) $P\left[\frac{1}{3} < X < \frac{1}{2}\right]$ (iii) $Mean\ of\ X$	**06**	**L2**	**CO3**
	b	The length of a telephone conversation has an exponential distribution with a mean of 3 minutes. Find the probability that a call i. ends in less than 3 minutes ii. Takes between 3 and 5 minutes	**07**	**L3**	**CO3**
	c	In a distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution.	**07**	**L3**	**CO3**
Module-4					
7	a	Find an optimal solution to the following LPP by computing all possible basic solutions and then finding one that maximizes the objective function. $Maximize\ Z = 2x_1 + 3x_2 + 4x_3 + 7x_4$, subject to: $2x_1 + 3x_2 - x_3 + 4x_4 = 8$ $x_1 - 2x_2 + 6x_3 - 7x_4 = -3$ $x_1, x_2, x_3, x_4 \ge 0$	**10**	**L3**	**CO4**
	b	Using the Simplex method to solve the L.P.P $Maximize\ Z = 3x_1 + 2x_2$, subject to: $2x_1 + x_2 \le 40$ $x_1 + x_2 \le 24$ $2x_1 + 3x_2 \le 60$ $x_1, x_2 \ge 0$	**10**	**L3**	**CO4**
OR					
8	a	Define the following terms A Linear Programming Problem, Basic solution, Basic feasible solution, Optimal solution, artificial variables of an LPP	**10**	**L3**	**CO4**
	b	Solve the LPP by the two-Phase method $Maximize\ Z = 5x_1 + 8x_2$, subject to: $3x_1 + 2x_2 \ge 3$ $x_1 + 4x_2 \ge 4$ $x_1 + x_2 \le 5$ $x_1, x_2 \ge$	**10**	**L3**	**CO4**

Module-5

9 a Find an initial basic feasible solution by Vogel's method to the following transportation problem. **06** **L3** **CO5**

		Destination					
		A	B	C	D	E	Availability
Source	I	3	4	6	8	8	**20**
	II	2	10	1	5	30	**30**
	III	7	11	20	40	15	**15**
	IV	2	1	9	14	18	**13**
	Requirements	**40**	**6**	**8**	**18**	**6**	

b Four jobs are to be done on four different machines. The cost (in rupees) of producing i^{th} job on the j^{th} machine is given below **07** **L3** **CO5**

		Machines			
		M_1	M_2	M_3	M_4
Jobs	J_1	15	11	13	**15**
	J_2	17	12	12	**13**
	J_3	14	15	10	**14**
	J_4	**16**	**13**	**11**	**17**

Assign the jobs to different machines so as to minimize the total cost

OR

10 a A company has three cement factories located in cities 1, 2, 3 which supply cement to four projects located in towns 1, 2, 3 4, each plant can supply 6, 1, 10 truckloads of cement daily respectively and daily cement requirements of the projects are respectively 7, 5, 3, 2 truck loads. The transport costs per truckload of cement (in hundreds of rupees) from each plant to each project site are as follows. **10** **L3** **CO5**

		Project sites				
		1	2	3	4	
Factories	1	2	3	11	7	
	2	1	0	6	1	
	3	5	8	15	9	

Determine the optimal distribution for the company so as to minimize the total transportation cost

b A car hire company has one car at each of five depots a, b, c, d and e. a custom R requires a car in each town, namely A,B,C,D and E. Distance (in kms) between depots and towns are given in the following distance matrix. **10** **L3** **CO5**

	a	b	c	d	e
A	160	130	175	190	200
B	135	120	130	160	175
C	140	110	155	170	185
D	50	50	80	80	110
E	55	35	70	80	105

How should cars be assigned to customers so as to minimize the distance travelled?

Bloom's Taxonomy Levels	**Lower-order thinking skills**		
	Remembering (knowledge): L_1	Understanding (Comprehension): L_2	Applying (Application): L_3
	Higher-order thinking skills		
	Analyzing (Analysis): L_4	Valuating (Evaluation): L_5	Creating (Synthesis): L_6