Model Question Paper 18AI56

Fifth Semester B.E. Degree Examination, March 2022

Mathematics for Machine Learning

Time: 3 hours **Max. Marks: 100**

Note: Answer any FIVE full questions, choosing ONE full question from each Module.

Module-1

1. a) Define Vector Space. Write the polynomial $f(t)=at^2+bt+c$ as a linear combination of the polynomials $p_1=(t-1)^2$, $p_2=t-1$ and $p_3=1$. **(06 Marks)**

   b) Suppose $u=(1,-3,4)$ and $v=(3,4,7)$. Find

   (i) $d(u,v)$, the distance between the vectors u and v.

   (ii) The angle between the vectors u and v.

   (iii) The projection of u onto v. **(06 Marks)**

   c) For what values of λ and μ the system of linear equations

   $$x+y+z=6$$
   $$x+2y+5z=10$$
   $$2x+3y+\lambda z=\mu$$

   have (i) no solution (ii) unique solution (iii) more than one solution. (iv) Also find the solution for $\lambda=2$ and $\mu=8$. **(08 Marks)**

OR

2. a) Define Inner Product Space.

   Consider $f(t)=3t-5$ and $g(t)=t^2$ in the polynomial space $P(t)$ with inner product $\langle f,g\rangle=\int_0^1 f(t)g(t)dt$. Find $\langle f,g\rangle$, $\|f\|$ and $\|g\|$. **(06 Marks)**

   b) Find the dimension and a basis for the general solution W of the following homogeneous system

   $$2x_1+4x_2-5x_3+3x_4=0$$
   $$3x_1+6x_2-7x_3+4x_4=0$$
   $$5x_1+10x_2-11x_3+6x_4=0$$ **(08 Marks)**

c) Determine whether or not W is a subspace of R^3 where $W=\{(a,b,c): a=2b=3c\}$.

Suppose the vectors u, v, w are linearly independent. Show that the vectors $u+v, u-v, u-2v+w$ are also linearly independent. **(06 Marks)**

Module-2

3. a) Let W be the subspace of R^4 orthogonal to $u_1=(1, 1, 2, 2)$ and $u_2=(0,1, 2,-1)$. Find (i) an orthogonal basis of W. (ii) an orthonormal basis of W. **(06 Marks)**

b) Let W be the subspace of R^5 spanned by $u=(1,2,3,-1,2)$ and $v=(2,4,7, 2,-1)$. Find a basis of the orthogonal complement $W^{\perp}$ of W. **(06 Marks)**

c) Let S consists of the following vectors in R^4 :

$u_1=(1, 1, 0, -1)$ $u_2=(1, 2, 1, 3)$ $u_3=(1, 1, -9, 2)$ and $u_4=(16, -13, 1, 3)$

(i) Show that S is orthogonal and a basis of R^4. (ii) Find the coordinates of an arbitrary vector $v=(a, b, c, d)$ relative to the basis S. **(08 Marks)**

OR

4. a) Find the eigen values and eigen vectors of the matrix $A=\begin{bmatrix}3 & 1 & 4\\ 0 & 2 & 6\\ 0 & 0 & 5\end{bmatrix}$. **(06 Marks)**

b) Find the matrix P which transforms the matrix $A=\begin{bmatrix}1 & 1 & 3\\ 1 & 5 & 1\\ 3 & 1 & 1\end{bmatrix}$ to the diagonal form. **(06 Marks)**

c) Find the singular value decomposition of the matrix $A=\begin{bmatrix}1 & 0 & 1\\ -2 & 1 & 0\end{bmatrix}$. **(08 Marks)**

Module-3

5. a) If $u=x+y+z$, $v=x^2+y^2+z^2$, $w=xy+yz+zx$ then prove that *grad u*, *grad v*, *grad w* are coplanar. **(06 Marks)**

b) A particle moves along the curve $\vec{r}=(t^3-4t)\hat{i}+(t^2+4t)\hat{j}+(8t^2-3t^3)\hat{k}$, where t is the time. Find the magnitude of the tangential components of its acceleration at $t=2$. **(06 Marks)**

c) Find the rate of change of $\phi(x,y,z)=xyz$ in the direction normal to the surface $x^2y+y^2x+yz^2=3$ at the point (1, 1, 1). **(08 Marks)**

OR

6. a) Find the values of the constants λ and μ such that the surfaces $\lambda x^2-\mu yz=(\lambda+2)x$ and $4x^2y+z^3=4$ intersect orthogonally at the point (1, -1, 2)? **(06 Marks)**

b) Define gradient of a vector valued function.

Consider the function $h:R\to R,\ h(t)=(f\circ g)(t)$ with $f:R^2\to R$ and $g:R\to R^2$.

If $f(x)=\exp(x_1x_2{}^2),\ x=\begin{bmatrix}x_1\\x_2\end{bmatrix}=g(t)=\begin{bmatrix}t\cos t\\t\sin t\end{bmatrix}$, then compute gradient of h with respect to t. **(08 Marks)**

c) Define Jacobian of a vector-valued function.

Suppose a vector-valued function $f:R^3\to R^3$ is defined as

$f(x_1,\ x_2,\ x_3)=\left(3x_1x_2+x_1x_3,\ 2x_1-4x_3{}^2,\ (x_1+x_2+2x_3)^2\right)$. Find the Jacobian of f at the point $(2,1,-1)$. **(06 Marks)**

Module-4

7. a) Let A and B be two events, not mutually exclusive, connected with a random experiment E. If $P(A)=1/4, P(B)=2/5$ and $P(A\cup B)=1/2$, find the values of the following probabilities: $(a)P(A\cap B), (b)P(A\cap B'), (c)P(A'\cup B')$. **(06 Marks)**

b) Suppose a function is defined as follows:

$$f(x)=e^{-x},\quad x\geq 0$$
$$=0,\quad x<0$$

(i) Is the function defined above, a density function?

(ii) If so, determine the probability that the variate having this density will fall in the interval (1, 2)?

(iii) Also find the cumulative probability function $F(2)$? **(06 Marks)**

c) A company has two plants to manufacture scooters. Plant I manufactures 80% of the scooters and Plant II manufactures 20%. At Plant I, 85 out of 100 scooters are rated standard quality or better. At Plant II, only 65 out of 100 scooters are rated standard quality or better. (a) What is the probability that a scooter selected at random came from Plant I if it is known that the scooter is of standard quality? (b) What is the probability the scooter came from Plant II if it is known that the scooter is of standard quality? **(08 Marks)**

OR

8. a) Determine the Binomial distribution for which Mean = 2 × Variance and Mean + Variance = 3. **(06 Marks)**

b) The probability distribution of a random variable x is given below.

x	-2	-1	0	1	2
$p(x)$	0.2	0.1	0.3	0.3	0.1

Find (a) $E(x)$ (b) $V(x)$ (c) $E(2x-3)$ (d) $V(2x-3)$. **(06 Marks)**

c) Suppose there is an outbreak of an epidemic (for example COVID-19) in a particular city and the government has decided to vaccinate the people residing in that city as a precautionary measure. Now, it is observed that, while vaccinating the people, the

probability that an individual suffers due to a bad reaction from a certain injection is 0.001. Then determine the probability that out of 2000 individuals
(i) Exactly 3 people will suffer due to a bad reaction, (ii) More than 2 people will suffer due to a bad reaction, (iii) No one will suffer due to a bad reaction. **(08 Marks)**

Module-5

9. a) What is Gradient Descent. With the help of gradient descent find the point at which the function $f(x_1, x_2) = x_1^2 - 2x_1x_2 + 2x_2^2 + 2x_1$ has optimal solution. **(06 Marks)**

   b) What do you mean by constrained optimization? Maximize the utility function $f(x, y) = xy$ subject to the constraint $g(x, y) = x + 4y = 240$. Here the price of per unit x is 1, the price of y is 4 and the budget available to buy x and y is 240. Solve the problem using constrained optimization. **(06 Marks)**

   c) Given that $x + y + z = \alpha$ (where α is a constant). Find the extremum value of the function $f(x, y, z) = x^m y^n z^p$. **(08 Marks)**

OR

10. a) Define a convex function. Determine whether the function $f(x) = x\log_2 x$ is convex or not for $x > 0$. **(08 Marks)**

    b) Using Lagrange's multiplier, find the dimensions of the rectangular box, open at the top, of maximum capacity, whose surface area is 432 sq cm. **(06 Marks)**

    c) Prove that a non-negative weighted sum of convex functions is always convex. **(06 Marks)**

Table showing the Bloom's Taxonomy Level, Course Outcome and Programme Outcome

Question		Bloom's Taxonomy Level attached	Course Outcome	Programme Outcome
Q.1	(a)	L1	CO1	PO1
	(b)	L1	CO1	PO2
	(c)	L2	CO1	PO3
Q.2	(a)	L1	CO1	PO2
	(b)	L1	CO1	PO3
	(c)	L2	CO1	PO2
Q.3	(a)	L1	CO1	PO3
	(b)	L1	CO1	PO2
	(c)	L1	CO1	PO3
Q.4	(a)	L1	CO1	PO2
	(b)	L1	CO1	PO1
	(c)	L2	CO1	PO1
Q.5	(a)	L1	CO2	PO2
	(b)	L1	CO2	PO2
	(c)	L1	CO2	PO1
Q.6	(a)	L1	CO2	PO3
	(b)	L1	CO2	PO1
	(c)	L2	CO2	PO1
Q.7	(a)	L1	CO3	PO1
	(b)	L1	CO3	PO2
	(c)	L1	CO3	PO2
Q.8	(a)	L1	CO3	PO1
	(b)	L1	CO3	PO2
	(c)	L2	CO3	PO2
Q.9	(a)	L1	CO4	PO1
	(b)	L1	CO4	PO2
	(c)	L2	CO4	PO1
Q.10	(a)	L1	CO4	PO1
	(b)	L1	CO4	PO1
	(c)	L1	CO4	PO2

Bloom's Taxonomy Levels	**Lower order thinking skills**		
	Remembering (knowledge): L_1	Understanding Comprehension): L_2	Applying (Application): L_3
	Higher order thinking skills		
	Analyzing (Analysis): L_4	Valuating (Evaluation): L_5	Creating (Synthesis): L_6