Model Question Paper Set - 1 with effect from 2022(CBCS Scheme)

USN | | | | | | | | | | |

Fourth Semester B.E Degree Examination

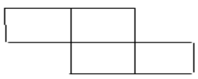
DISCRETE MATHEMATICAL STRUCTURES (BCS405A)

TIME: 03Hours **Max.Marks:100**

Note:

1. Answer any **FIVE** full questions, choosing at least **ONE** question from each **MODULE**
2. M: Marks, L: RBT levels, C: Course outcomes.

		Module – 1	M	L	C
Q.1	a	Show that the compound proposition $\left[(p \leftrightarrow q) \wedge (q \leftrightarrow r) \wedge (r \leftrightarrow p)\right] \Leftrightarrow \left[(p \rightarrow q) \wedge (q \rightarrow r) \wedge (r \rightarrow p)\right]$ for primitive statements p, q, r is logically equivalence.	6	L2	CO1
	b	Establish the validity of the following argument using the Rules of Inference: $\{p \wedge (p \rightarrow q) \wedge (s \vee r) \wedge (r \rightarrow \sim q)\} \longrightarrow (s \vee t)$.	7	L2	CO1
	c	For the universe of all integers, let p(x), q(x), r(x), s(x) and t(x) denote the following open statements: p(x): x>0, q(x): x is even, r(x): x is a perfect square, s(x): x is divisible by 3, t(x): x is divisible by 7. Write the following statements in symbolic form: i) At least one integer is even. ii) There exists a positive integer that is even. iii) If x is even, then x is not divisible by 3. iv) No even integer is divisible by 7. v) There exists even integer divisible by 3.	7	L1	CO1
		OR			
Q.2	a	Define a tautology. Prove that, for any propositions p, q, r the compound propositions, $\{(p \rightarrow q) \wedge (q \rightarrow r)\} \rightarrow \{(p \rightarrow r)\}$ is tautology.	6	L2	CO1
	b	Prove the following using laws of logic: $p \rightarrow (q \rightarrow r) \Leftrightarrow (p \wedge q) \rightarrow r$.	7	L2	CO1
	c	Give i) direct proof ii) indirect proof iii) proof by contradiction for the following statement: "if n is an odd integer then n+9 is an even integer".	7	L3	CO1
		Module – 2			
Q.3	a	Prove that $1^2 + 3^2 + 5^2 + \cdots . + (2n-1)^2 = \frac{n(2n+1)(2n-1)}{3}$ by Mathematical Induction.	6	L2	CO2
	b	Let $a_0 = 1, a_1 = 2, a_2 = 3$ and $a_n = a_{n-1} + a_{n-2} + a_{n-3}$ for $n \geq 3$. Prove that $a_n \leq 3^n \ \forall \ n \in z^+$.	7	L2	CO2
	c	Find the number of ways of arrangement of the letters of the word 'TALLAHASSEE' which have no adjacent A's.	7	L2	CO2
		OR			
Q.4	a	Determine the coefficient of xyz^2 in the expansion of $(2x - y - z)^4$.	6	L2	CO2
	b	In how many ways one can distribute 8 identical marbles in 4 distinct containers so that i) no container is empty ii) the fourth container has an odd number of marbles in it.	7	L2	CO2
	c	How many positive integers n can we form using the digits 3,4,4,5,5,6,7 if we want n to exceed 5,000,000?	7	L2	CO2
		Module – 3			
Q.5	a	Let f: R → R be defined by, $f(x) = \begin{cases} 3x - 5 \ if \ x > 0 \\ 1 - 3x \ if \ x \leq 0 \end{cases}$. Find $f^{-1}(0), f^{-1}(1), f^{-1}(-1), f^{-1}(3), f^{-1}(6), \ f^{-1}([-6,5])$ and $f^{-1}([-5,5])$	6	L2	CO3

	b	State Pigeon hole principle. Prove that in any set of 29 persons; at least 5 persons have been born on the same day of the week.	7	L2	CO3
	c	Let A={1,2,3,4,6} and 'R' be a relation on 'A' defined by aRb if and only if "a is multiple of b" represent the relation 'R' as a matrix, draw the diagraph and relation R.	7	L2	CO3
		OR			
Q.6	a	If $f{:}A\to B$, $g{:}B\to C$, $h{:}C\to D$ are three functions, then Prove that h∘(g∘f)=(h∘g)∘f.	6	L2	CO3
	b	Show that if n+1 numbers are chosen from 1 to 2n then at least one pair add to 2n+1.	7	L2	CO3
	c	Draw the Hasse diagram representing the positive divisors of 72.	7	L2	CO3
		Module – 4			
Q.7	a	In how many ways the 26 letters of English alphabet are permuted so that none of the pattern's CAR, DOG, PUN or BYTE occurs?	6	L2	CO4
	b	Define Derangement. In how many ways can each of 10 people select a left glove and a right glove out of a total of 10 pairs of gloves so that no person selects a matching pair of gloves?	7	L2	CO4
	c	Solve the recurrence relation: $C_n = 3C_{n-1} - 2C_{n-2}, for\ n \geq 2$, given $C_1 = 5, C_2 = 3$.	7	L3	CO4
		OR			
Q.8	a	In how many ways one can arrange the letters of the word **CORRESPONDENTS** so that there are i) exactly 2 pairs of consecutive identical letters? ii) at least 3 pairs of consecutive identical letters? iii) no pair of consecutive identical letters?	6	L2	CO4
	b	Find the rook polynomial for the chess board as shown in the figure 	7	L2	CO4
	c	Solve the recurrence relation $a_{n+2} - 3a_{n+1} + 2a_n = 0, a_0 = 1, a_1 = 6$.	7	L3	CO4
		Module – 5			
Q.9	a	If H, K are subgroups of a group G, prove that $H \cap K$ is also a subgroup of G. Is $H \cup K$ a subgroup of G.	6	L2	CO5
	b	Define Klein 4 group. Verify $A = \{1, 3, 5, 7\}$ is a klein 4 group.	7	L2	CO5
	c	State and prove Lagrange's Theorem.	7	L2	CO5
		OR			
Q.10	a	Show that i) the identity of G is unique. ii) the inverse of each element of G is Unique.	6	L3	CO5
	b	Show that $(A, \cdot)$ is an abelian group where $A = \{a \in Q \mid a \neq -1\}$ and for any $a, b \in A, a\,.\,b = a + b + ab$.	7	L3	CO5
	c	Let $G = S_4$, for $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$, find the subgroup $H = \langle \alpha \rangle$. Determine the left cosets of H in G.	7	L3	CO5

Model Question Paper Set - 2 with effect from 2022(CBCS Scheme)

USN

Fourth Semester B.E Degree Examination

DISCRETE MATHEMATICAL STRUCTURES (BCS405A)

TIME:03Hours **Max.Marks:100**

Note:

1. Answer any **FIVE** full questions, choosing at least **ONE** question from each **MODULE**
2. M: Marks, L: RBT levels, C: Course outcomes.

		Module - 1	M	L	C
Q.1	a	Define tautology. Show that $[(p\vee q)\wedge(p\to r)\wedge(q\to r)]\to r$ is a tautology by constructing the truth table.	6	L1	CO1
	b	Prove the following using the laws of logic $[\neg p\wedge(\neg q\wedge r)]\vee[(q\wedge r)\vee(p\wedge r)]\Leftrightarrow r$.	7	L2	CO1
	c	For any two odd integers m and n, show that (i) m+n is even (ii) mn is odd.	7	L2	CO1
		OR			
Q.2	a	Define i) an open statement ii) Quantifiers	6	L2	CO1
	b	Write the following argument in symbolic form and then establish the validity If A gets the Supervisor's position and works hard, then he will get a raise. If he gets a raise, then he will buy a car. He has not purchased a car. Therefore he did not get the Supervisor's position or he did not work hard.	7	L1	CO1
	c	For the following statements, the universe comprises all non-zero integers. Determine the truth value of each statement. a) $\exists\, x \exists\, y\, [xy=1]$ b) $\exists\, x\forall y\, [xy=1]$ c) $\forall x \exists\, y\, [xy=1]$ d) $\exists\, x\exists\, y[(2x+y=5)\wedge(x-3y=-8)]$ e) $\exists\, x\exists\, y[(3x-y=7)\wedge(2x+4y=3)]$	7	L2	CO1
		Module – 2			
Q.3	a	Define the well ordering principle. By Mathematical Induction, Prove that $(\boldsymbol{n}!)\geq \mathbf{2n-1}$ for all integers $\boldsymbol{n}\geq\mathbf{1}$.	6	L2	CO2
	b	Prove that every positive integer n≥24 can be written as a sum of 5's and/or 7's.	7	L3	CO2
	c	How many positive integers n , can we form using the digits 3,4,4,5,5,6,7, if we want n to exceed 5,000,000.	7	L1	CO2
		OR			
Q.4	a	By Mathematical Induction Prove that $1.3+2.4+\cdots\ldots\ldots..+n(n+2)=\frac{n(n+1)(2n+7)}{6}$.	6	L1	CO2
	b	Find the number of permutations of the letters of the word MASSASAUGA. In how many of these all four $A's$ are together? How many of them begin with S?	7	L2	CO2
	c	i) Obtain the Coefficient of a^5b^2 in the expansion of $(2a-3b)^7$ ii) Using the Binomial theorem find the coefficient of x^5y^2 in	7	L1	CO2

		the expansion of $(x+y)^7$.			
Module – 3					
Q.5	**a**	State Pigeon –hole principle. Prove that if any number from1 to 8 are chosen then two of them will have their sum as 9.	**6**	**L1**	**CO3**
	b	Let $f: R \to R$ be defined by ,$f(x) = \begin{Bmatrix} 3x-5, & if\ x > 0 \\ 1-3x, & if x \leq 0 \end{Bmatrix}$ $find, f^{-1}([-6,5])\ and\ f^{-1}([-5,5])$.	**7**	**L1**	**CO3**
	c	Let $A = B = C = R$, and $f: A \to B$ and $g: B \to C$ be defined by $f(a) = 2a + 1, \quad g(b) = \frac{1}{3}b, \forall a \in A, \forall b \in B.$ Compute gof and show that $gof$is invertible. What is $(g\ o\ f)^{-1}$?	**7**	**L2**	**CO3**
OR					
Q.6	**a**	Let f and g be functions from R to R defined by $f(x) = ax + b$ and $g(x) = 1 - x + x^2$, If $(g\ o\ f)(x) = 9x^2 - 9x + 3$ determine a and b.	**6**	**L3**	**CO3**
	b	Draw the Hasse (POSET) diagram which represents positive divisors of 36.	**7**	**L2**	**CO3**
	c	Let $A = \{1,2,3,4,6\}$ and R be a relation on A defined by aRb if and only if "a is a multiple of b". Write down the relation R, relation matrix $M(R)$ And draw its diagraph. List out its in degree and out degree.	**7**	**L2**	**CO3**
Module – 4					
Q.7	**a**	Determine the number of positive integers n such that $1 \leq n \leq 100$ and n is not divisible by 2, 3, or 5	**6**	**L2**	**CO4**
	b	In how many ways can the 26 letters of the English alphabet be permuted so that none of the patterns CAR, DOG, PUN or BYTE occurs?	**7**	**L2**	**CO4**
	c	Solve the recurrence relation $a_n = na_{n-1}$ where n ≥ 1and $a_0 = 1$.	**7**	**L2**	**CO4**
OR					
Q.8	**a**	In how many ways 5 number of a's, 4 number of b's and 3 number of c's can be arranged so that all the identical letters are not in a single block?	**6**	**L3**	**CO4**
	b	Five teachers T_1, T_2, T_3, T_4, T_5 are to be made class teachers for five classes, C_1, C_2, C_3, C_4, C_5, one teacher for each class. $T_1$and T_2 do not wish to become the class teachers for C_1or C_2, T_3 and T_4 for C_4 or C_5 , and $T_5$for C_3 or C_4 or C_5 .In how many ways can the teachers be assigned the work (without displeasing any teacher	**7**	**L2**	**CO4**
	c	Solve the recurrence relation $F_{n+2} = F_{n+1} + F_n$where n ≥ 0 and $F_0 = 0, F_1 = 1$.	**7**	**L2**	**CO4**
Module – 5					
Q.9	**a**	Define group. Show that fourth roots of unity is an abelian group.	**6**	**L2**	**CO5**
	b	If G be a set of all non-zero real numbers and let a* b = ab/2 then show that (G,*) is an abelian group.	**7**	**L2**	**CO5**
	c	Define Klein 4-group. And if A = { e,a,b,c} then show that this is a Klein -4 group	**7**	**L1**	**CO5**
OR					
Q.10	**a**	Define Cyclic group and show that (G,8) whose multiplication table is as given below is Cyclic	**6**	**L2**	**CO5**

*	a	b	c	d	e	f
a	a	b	c	d	e	f
b	b	c	d	e	f	a
c	c	d	e	f	a	b
d	d	e	f	a	b	c
e	e	f	a	b	c	d
f	f	a	b	c	d	e

b	State and prove Lagrange's theorem	7	L1	CO5
c	If G be a group with subgroup H and K. If $\|G\|$ = 660 and $\|K\|$ = 66 and K C H C G and find the possible value for $\|H\|$	7	L2	CO5