# Model Question Paper-II with effect from 2022(CBCS Scheme)

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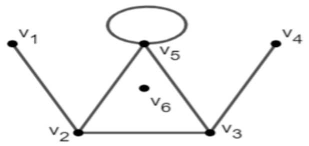
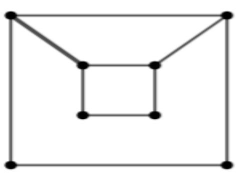
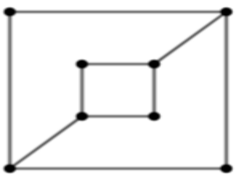
## Fourth Semester B.E Degree Examination

## GRAPH THEORY (BCS405B)

**TIME:03Hours** **Max.Marks:100**

Note:

1. Answer any **FIVE** full questions, choosing at least **ONE** question from each **MODULE**
2. VTU Formula Hand Book is Permitted
3. M: Marks, L: RBT levels, C: Course outcomes.

| | | Module – 1 | M | L | C |
|---|---|---|---|---|---|
| Q.1 | a | Consider the following graph $G$.<br>(i) What type of a graph is $G$?<br>(ii) Find the pendant vertices in $G$.<br>(iii) How many components are there in $G$?<br>(iv) Find the minimum degree, $\delta(G)$ in $G$.<br>(v) Find the average degree, $d(G)$ of the graph $G$.<br>Draw two vertex disjoint subgraphs of $G$.<br><br><br>G | 6 | L2 | CO1 |
| | b | Show that the number of vertices of odd degree in a graph is always even. | 7 | L3 | CO1 |
| | c | Show that the maximum number of edges in a simple graph with $n$ vertices is $\frac{n(n-1)}{2}$. | 7 | L3 | CO1 |
| | | OR | | | |
| Q.2 | a | Distinguish between Complete graph and Complete Bipartite graph. | 6 | L2 | CO1 |
| | b | Verify whether the following graphs are isomorphic or not.<br><br>$G_1$<br><br>$G_2$ | 7 | L2 | CO1 |
| | c | Show that a simple graph with $n$ vertices and $k$ components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges. | 7 | L3 | CO1 |
| | | Module – 2 | | | |
| Q.3 | a | By specifying the walk, draw two Euler graphs and an Unicursal graph. | 6 | L2 | CO2 |
| | b | If all the vertices in a connected graph $G$ are of even degree, then show that $G$ is Eulerian. | 7 | L3 | CO2 |
| | c | Define Hamiltonian cycle. How many edge-disjoint Hamiltonian cycles exist in a complete graph with 5 vertices? Draw the graph to show and specify the cycle. | 7 | L1 | CO2 |

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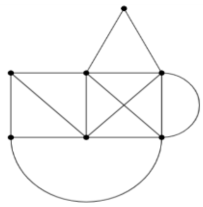
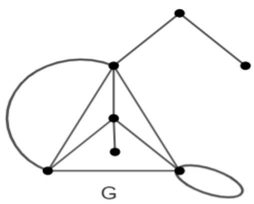
| | | **OR** | | | |
| **Q.4** | **a** | Define Hamiltonian graph. By specifying the walk, draw a graph that has a Hamiltonian path but does not have a Hamiltonian circuit. | **6** | **L1** | **CO2** |
| | **b** | Show that a connected graph $G$ has an Eulerian trail if and only if there are exactly two vertices of odd degree in $G$. | **7** | **L3** | **CO2** |
| | **c** | (i) Prove that in any digraph the sum of the in-degrees of all vertices is equal to the sum of their out-degrees; and this sum is equal to the number of edges in the digraph.<br>(ii) Draw a complete symmetric digraph and a complete asymmetric digraph with 4 vertices. | **7** | **L3** | **CO2** |
| | | **Module – 3** | | | |
| **Q.5** | **a** | (i) Show that the number of vertices in a binary tree is always odd.<br>(ii) Find the number of pendant vertices in a binary tree of order $n$. | **6** | **L3** | **CO3** |
| | **b** | Prove that a connected graph G is a tree if and only if there is one and only one path between every pair of vertices. | **7** | **L3** | **CO3** |
| | **c** | Show that a tree with $n$ vertices has $n-1$ edges. | **7** | **L3** | **CO3** |
| | | **OR** | | | |
| **Q.6** | **a** | (i) Show that every connected graph contains a spanning tree.<br>(ii) Find the number of tree branches and chords in the following graph with 7 vertices and 14 edges.<br> | **6** | **L3** | **CO3** |
| | **b** | Define Fundamental Circuit. If $G$ is a graph with $n$ vertices and $q$ edges, then find the number of fundamental circuits in the graph. | **7** | **L1** | **CO3** |
| | **c** | Show that for any graph $G$, the vertex connectivity cannot exceed the edge connectivity and the edge connectivity cannot exceed the degree of the vertex with the smallest degree in $G$. | **7** | **L3** | **CO3** |
| | | **Module – 4** | | | |
| **Q.7** | **a** | (i) Define planar and non-planar graphs.<br>(ii) State Kuratowski's theorem. Draw Kuratowski's two graphs. | **6** | **L1** | **CO4** |
| | **b** | Show that a connected planar graph with $n$ vertices and $e$ edges has $e-n+2$ regions. | **7** | **L3** | **CO4** |
| | **c** | 1. Draw the geometric dual of the graph G.<br>2. Write down the adjacency matrix for the graph G.<br><br>G | **7** | **L2** | **CO4** |
| | | **OR** | | | |
| **Q.8** | **a** | If $G$ is a simple planar graph with at least three vertices, then show that (i) $e \leq 3n-6$. and (ii) $e \leq 2n-4$ ; if $G$ is triangle free. | **6** | **L3** | **CO4** |
| | **b** | (i) Show that Petersen graph is non-planar.<br>(ii) Let $G$ be a planar graph. Then prove that it contains a vertex of degree at most 5. | **7** | **L3** | **CO4** |
| | **c** | Write down the Path matrix and Circuit matrix for the given graph. | **7** | **L2** | **CO4** |

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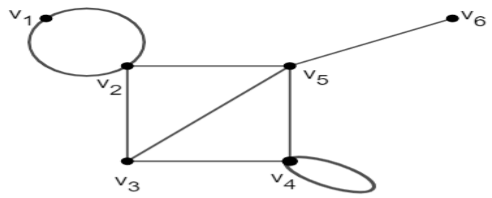
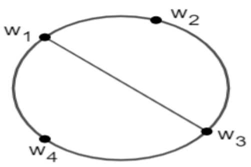
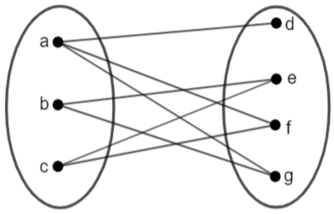
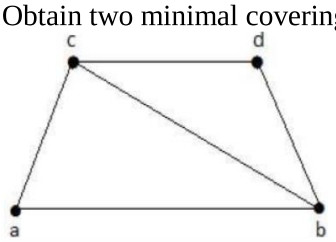
| | |   | | | |
| **Module – 5** | | | | | |
| **Q.9** | **a** | Prove that every tree with two or more vertices is 2-chromatic. | **6** | **L3** | **CO5** |
| | **b** | Define chromatic number of a graph. Find the chromatic polynomial and chromatic number for the given graph.   | **7** | **L1** | **CO5** |
| | **c** | Define Matching and complete matching. Obtain two complete matching from the given graph.   | **7** | **L1** | **CO5** |
| **OR** | | | | | |
| **Q.10** | **a** | Prove that an $n$ -vertex graph is a tree if and only if its chromatic polynomial is $P_n(\lambda) = \lambda(\lambda - 1)^{n-1}$. | **6** | **L3** | **CO5** |
| | **b** | Define Covering and minimal covering of a graph.<br>Obtain two minimal coverings from the given graph.   | **7** | **L1** | **CO5** |
| | **c** | State and prove Five color theorem. | **7** | **L2** | **CO5** |