Model Question Paper - I with effect from 2022(CBCS Scheme)

USN

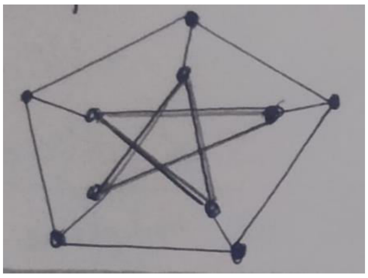
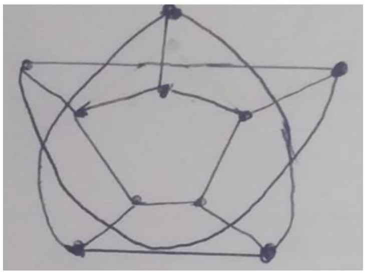
Fourth Semester B.E Degree Examination

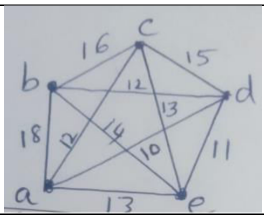
GRAPH THEORY (BCS405B)

TIME:03Hours **Max.Marks:100**

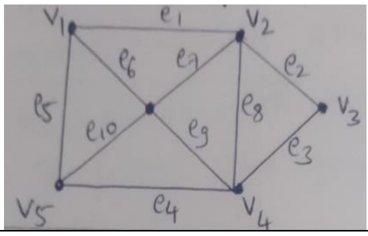
Note:

1. Answer any **FIVE** full questions, choosing at least **ONE** question from each **MODULE**
2. M: Marks, L: RBT levels, C: Course outcomes.

		Module - 1	M	L	C
Q.1	a	Define i) Walk ii) Path iii) Circuit with an example for each	6	L1	CO1
	b	If a connected graph G is decomposed into two subgraphs g_1 and g_2, then prove that there must be at least one vector common between g_1 and g_2.	7	L3	CO1
	c	Determine whether the following graphs are isomorphic or not.  	7	L3	CO1
		OR			
Q.2	a	Define i) Isomorphism ii) Subgraph iii) Pendent vertex with an example for each.	6	L1	CO1
	b	Prove that the number of vertices of odd degree in a graph is always even.	7	L3	CO1
	c	Explain any Five applications of graphs.	7	L3	CO1
		Module – 2			
Q.3	a	Prove that a connected graph G is an Euler graph if and only if all vertices of G are of even degree.	6	L3	CO2
	b	Define Hamiltonian circuit and Hamiltonian path. Give an example for each. Also, draw a graph that has a Hamiltonian path but not a Hamiltonian circuit.	7	L3	CO2
	c	Discuss about any four types of digraphs with suitable examples.	7	L3	CO2
		OR			
Q.4	a	In a complete graph with n vertices, prove that there are $\frac{n-1}{2}$ edge-disjoint Hamiltonian circuits, if n is an odd number ≥ 3	6	L3	CO2
	b	The weights in the graph given below represent the distances between cities. A salesman based at city ‘a’ would like to visit every other city exactly once and return to the home city, keeping his total travel to a minimum. What route should he take and how far will he travel?	7	L3	CO2

					
	c	Define Binary relation. Represent the relation R defined on $A = \{2, 3, 4, 6\}$ by the phrase 'is a factor of' in a digraph.	7	L3	CO2

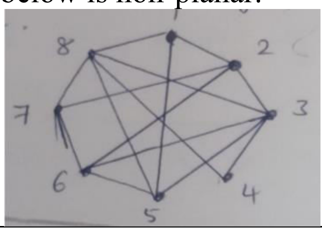
Module – 3

Q.5	a	For any Spanning tree of a connected graph with n vertices and e edges, prove that there are $n-1$ tree branches and $e-n+1$ chords. For the following graph, find two spanning trees and hence show that an edge that is a branch of one spanning tree can be a chord with respect to another spanning tree of same graph. 	6	L3	CO3
	b	Define vertex connectivity and edge connectivity. Give the relation between them.	7	L3	CO3
	c	Prove that every circuit has even number of edges in common with a cut-set.	7	L3	CO3

OR

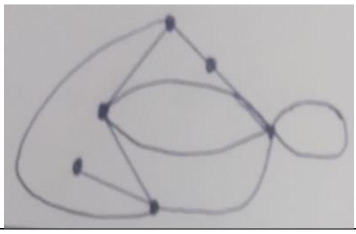
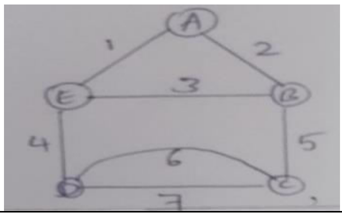
Q.6	a	Prove that there are at least two pendent vertices in a tree with two or more vertices.	6	L3	CO3
	b	Prove that the distance between any two spanning trees is a metric. Find two different minimum spanning trees of a graph with $\boldsymbol{V} = \{\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}\}$ described by $\boldsymbol{\varphi} = \begin{bmatrix} \boldsymbol{a} & \boldsymbol{b} & \boldsymbol{c} & \boldsymbol{d} & \boldsymbol{e} & \boldsymbol{f} \\ \{\mathbf{1},\mathbf{2}\} & \{\mathbf{1},\mathbf{2}\} & \{\mathbf{1},\mathbf{4}\} & \{\mathbf{2},\mathbf{3}\} & \{\mathbf{3},\mathbf{4}\} & \{\mathbf{3},\mathbf{4}\} \end{bmatrix}$ It has weights on its edges given by $\lambda = \begin{bmatrix} a & b & c & d & e & f \\ 3 & 2 & 1 & 2 & 4 & 2 \end{bmatrix}$	7	L3	CO3
	c	Prove that with respect to the given spanning tree T, a branch b_1 that determines a fundamental cut-set S is contained in every fundamental circuit associated with the chords in S and in no other.	7	L3	CO3

Module – 4

Q.7	a	State and prove Euler's formula that gives the number of regions in any planar graph.	6	L3	CO4
	b	Describe the steps to find adjacency matrix and incidence matrix for a directed graph with a simple example.	7	L3	CO4
	c	State Kuratowaki's Theorem and use it in order to prove the graph given below is non-planar. 	7	L3	CO4

OR

Q.8	a	Give two conditions for testing planarity of a given graph. Sketch a sample graph for planar graph and non-planar graph.	6	L3	CO4

	b	Draw the geometric dual of the following graph. 	7	L3	CO4
	c	Write the adjacency matrix and incidence matrix for the following graph. 	7	L3	CO4
Module – 5					
Q.9	**a**	Define Chromatic number. Prove that a graph with at least one edge is 2-chromatic if and only if it has no circuits of odd length.	6	L3	CO5
	b	Define chromatic polynomial and write the chromatic polynomial of a graph with n vertices.	7	L3	CO5
	c	State and prove Four-color Theorem.	7	L3	CO5
OR					
Q.10	**a**	Define i) Complete Matching ii) Minimal Covering. Give one example for each.	6	L1	CO5
	b	State and prove Five-color Theorem.	7	L3	CO5
	c	Write a note on Greedy coloring algorithm.	7	L3	CO5

Model Question Paper-II with effect from 2022(CBCS Scheme)

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Fourth Semester B.E Degree Examination

GRAPH THEORY (BCS405B)

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Note:

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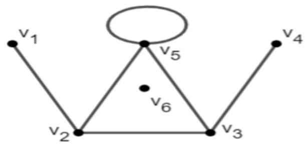
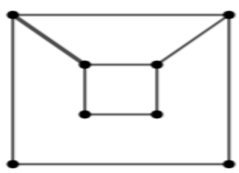
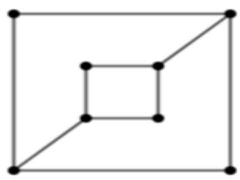
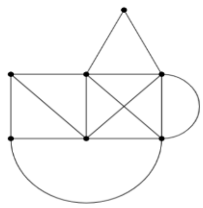
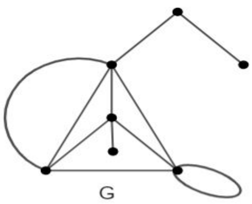
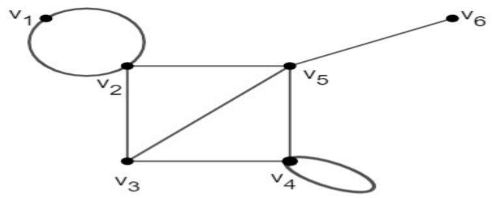
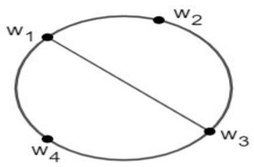
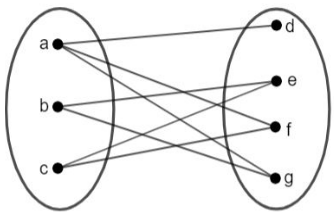
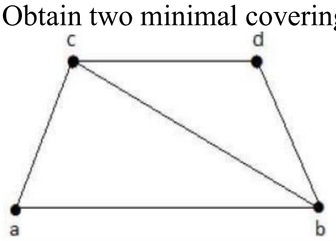
2. M: Marks, L: RBT levels, C: Course outcomes.

		Module – 1	M	L	C
Q.1	a	Consider the following graph G. (i) What type of a graph is G? (ii) Find the pendant vertices in G. (iii) How many components are there in G? (iv) Find the minimum degree, $\delta(G)$ in G. (v) Find the average degree, $d(G)$ of the graph G. Draw two vertex disjoint subgraphs of G.  G	6	L2	CO1
	b	Show that the number of vertices of odd degree in a graph is always even.	7	L3	CO1
	c	Show that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$.	7	L3	CO1
		OR			
Q.2	a	Distinguish between Complete graph and Complete Bipartite graph.	6	L2	CO1
	b	Verify whether the following graphs are isomorphic or not.  G_1  G_2	7	L2	CO1
	c	Show that a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.	7	L3	CO1
		Module – 2			
Q.3	a	By specifying the walk, draw two Euler graphs and an Unicursal graph.	6	L2	CO2
	b	If all the vertices in a connected graph G are of even degree, then show that G is Eulerian.	7	L3	CO2
	c	Define Hamiltonian cycle. How many edge-disjoint Hamiltonian cycles exist in a complete graph with 5 vertices? Draw the graph to show and specify the cycle.	7	L1	CO2
		OR			

Q.4	**a**	Define Hamiltonian graph. By specifying the walk, draw a graph that has a Hamiltonian path but does not have a Hamiltonian circuit.	**6**	**L1**	**CO2**
	b	Show that a connected graph G has an Eulerian trail if and only if there are exactly two vertices of odd degree in G.	**7**	**L3**	**CO2**
	c	(i) Prove that in any digraph the sum of the in-degrees of all vertices is equal to the sum of their out-degrees; and this sum is equal to the number of edges in the digraph. (ii) Draw a complete symmetric digraph and a complete asymmetric digraph with 4 vertices.	**7**	**L3**	**CO2**
		Module – 3			
Q.5	**a**	(i) Show that the number of vertices in a binary tree is always odd. (ii) Find the number of pendant vertices in a binary tree of order n.	**6**	**L3**	**CO3**
	b	Prove that a connected graph G is a tree if and only if there is one and only one path between every pair of vertices.	**7**	**L3**	**CO3**
	c	Show that a tree with n vertices has $n-1$ edges.	**7**	**L3**	**CO3**
		OR			
Q.6	**a**	(i) Show that every connected graph contains a spanning tree. (ii) Find the number of tree branches and chords in the following graph with 7 vertices and 14 edges. 	**6**	**L3**	**CO3**
	b	Define Fundamental Circuit. If G is a graph with n vertices and q edges, then find the number of fundamental circuits in the graph.	**7**	**L1**	**CO3**
	c	Show that for any graph G, the vertex connectivity cannot exceed the edge connectivity and the edge connectivity cannot exceed the degree of the vertex with the smallest degree in G.	**7**	**L3**	**CO3**
		Module – 4			
Q.7	**a**	(i) Define planar and non-planar graphs. (ii) State Kuratowski's theorem. Draw Kuratowski's two graphs.	**6**	**L1**	**CO4**
	b	Show that a connected planar graph with n vertices and e edges has $e-n+2$ regions.	**7**	**L3**	**CO4**
	c	1. Draw the geometric dual of the graph G. 2. Write down the adjacency matrix for the graph G.  	**7**	**L2**	**CO4**
		OR			
Q.8	**a**	If G is a simple planar graph with at least three vertices, then show that (i) $e \leq 3n-6$. and (ii) $e \leq 2n-4$; if G is triangle free.	**6**	**L3**	**CO4**
	b	(i) Show that Petersen graph is non-planar. (ii) Let G be a planar graph. Then prove that it contains a vertex of degree at most 5.	**7**	**L3**	**CO4**
	c	Write down the Path matrix and Circuit matrix for the given graph.	**7**	**L2**	**CO4**

					
		Module – 5			
Q.9	**a**	Prove that every tree with two or more vertices is 2-chromatic.	6	L3	CO5
	b	Define chromatic number of a graph. Find the chromatic polynomial and chromatic number for the given graph. 	7	L1	CO5
	c	Define Matching and complete matching. Obtain two complete matching from the given graph. 	7	L1	CO5
		OR			
Q.10	**a**	Prove that an n -vertex graph is a tree if and only if its chromatic polynomial is $P_n(\lambda) = \lambda(\lambda - 1)^{n-1}$.	6	L3	CO5
	b	Define Covering and minimal covering of a graph. Obtain two minimal coverings from the given graph. 	7	L1	CO5
	c	State and prove Five color theorem.	7	L2	CO5