Model Question Paper Set -1with effect from 2022(CBCS Scheme)

USN

Fourth Semester B.E Degree Examination

LINEAR ALGEBRA (BCS405D)

TIME:03Hours **Max.Marks:100**

Note:

1. Answer any **FIVE** full questions, choosing at least **ONE** question from each **MODULE.**
2. M: Marks, L: RBT levels, C: Course outcomes.

		Module - 1	M	L	C
Q.1	a	Show that set $V = \{a + b\sqrt{2} \mid a, b \in Q\}$, where Q is the set of all rational: field Q is a vector space, under usual addition and scalar multiplication.	6	L2	CO1
	b	Let $S = \{(1, -3,\ 2), (2, 4, 1), (1, 1, 1)\}$ be a subset of $V_3(R)$. Show that the vector $(3, -7, 6)$ is in L[S].	7	L2	CO1
	c	Show that the vectors $(1, 1, 2, 4), (2, -1, -5, 2), (1, -1, -4, 0)$ *and* $(2, 1, 1, 6)$ are linearly dependent in R^4 and extract a linearly independent subset. Also find the dimension and a basis of the subspace spanned by them.	7	L3	CO1
		OR			
Q.2	a	Prove that a non-empty subset W is a subspace of a vector space V over F, if and only if $c_1\, \alpha + c_2\, \beta\ \in W, \forall\, \alpha, \beta\ \in W,\ c_1, c_2\ \in F$. Show that the subset $W = \{(x, y, z) \mid x + y + z = 0\}$ of the vector space $V_3(R)$ is a subspace of $V_3(R)$.	6	L2	CO1
	b	Verify the set $S = \{(1, 2, 1), (-1, 1, 0), (5, -1, 2)\}$ is linearly dependent or not.	7	L2	CO1
	c	Find the basis and dimension of the subspace spanned by the subset $S = \left\{\begin{bmatrix} 1 & -5 \\ -4 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ -1 & 5 \end{bmatrix}, \begin{bmatrix} 2 & -4 \\ -5 & 7 \end{bmatrix}, \begin{bmatrix} 1 & -7 \\ -5 & 1 \end{bmatrix}\right\}$ of the vector space of all 2 x 2 matrices over R.	7	L3	CO1
		Module – 2			
Q.3	a	Find the linear transformation of $T: V_2(R) \to V_2(R)$ such that $T(1, 1) = (0, 1)$ and $T(-1, 1) = (3, 2)$.	6	L3	CO2
	b	Find the matrix of the linear transformation $T: V_2(R) \to V_3(R)$ defined by $T(x, y) = (2y - x,\ y, 3y - 3x)$ relative to bases $B_1 = \{(1, 1), (-1, 1)\}$ and $B_2 = \{(1, 1, 1), (1, -1, 1), (0, 0, 1)\}$.	7	L3	CO2
	c	Find the range, nullspace, rank and nullity of the linear transformation. $T: V_3(R) \to V_2(R)$ defined by $T(x, y, z) = (y - x,\ y - z)$ and verify also verify Rank-nullity theorem.	7	L3	CO2
		OR			
Q.4	a	Verify the transformation $T: V_2(R) \to V_2(R)$ defined by $T(x, y) = (3x + 2y, 3x - 4y)$ is linear or not.	6	L2	CO2

	b	Given the matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ -1 & 3 \end{bmatrix}$ find the linear transformation $T: V_2(R) \to V_3(R)$ relative to the bases $B_1 = \{(1,2), (-2,1)\}$ and $B_2 = \{(1,-1,-1), (1,2,3), (-1,0,2)\}$.	7	L2	CO2
	c	Show that the linear map $T: V_3 \to V_3$ defined by $T(e_1) = e_1 + e_2, T(e_2) = e_2 + e_3, T(e_3) = e_1 + e_2 + e_3$ is non-singular and find its inverse.	7	L2	CO2

Module – 3

Q.5	a	Determine the Eigen values and Eigen vectors of $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 4 \end{bmatrix}$.	6	L3	CO3
	b	Verify the Cayley's Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & -2 & 4 \end{bmatrix}$	7	L2	CO3
	c	Let V be a vector space of dimension 6 over R and let T be a linear operator whose minimal polynomial is $m(x) = (x^2 - x + 3)(x - 2)^2$. Find the rational canonical form of T.	7	L2	CO3

OR

Q.6	a	Find the eigen values and eigen vectors of $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.	6	L3	CO3
	b	Determine all the possible Jordan canonical forms for a linear operator $T: V \to V$ whose characteristics polynomial is $\Delta(x) = (x-2)^3(x-5)^2$	7	L3	CO3
	c	Find the characteristics equation for the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ and hence find A^{-1}.	7	L3	CO3

Module – 4

Q.7	a	Let V be a vector space of real continuous functions on the interval $0 \leq t \leq 1$ with inner product defined by $\langle f, g \rangle = \int_0^1 f(t) g(t)\, dt$ and the polynomial $f(t) = t + 2,\ g(t) = 3t - 2, h(t) = t^2 - 2t - 3$. Find $\langle f, g \rangle, \langle f, h \rangle, \|f\|, \|g\|$.	6	L2	CO4
	b	Construct an orthogonal basis of $A = \begin{bmatrix} 1 & 3 & 5 \\ 1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 3 & 3 \end{bmatrix}$ by Gram-Schmidt method.	7	L3	CO4
	c	Find a least square solution of the system of equation Ax=b. where $A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}, b = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$.	7	L3	CO4

OR

Q.8	a	If W is a subspace of a real inner product space V, prove that $W^{\perp}$ is a subspace of V.	6	L2	CO4

	b	Find an orthogonal basis for the vector space $V_3(R)$ by applying the Gram-Schmidth orthogonalization process to the vectors $(3, 0, 4), (-1, 0\ 7), (2, 9, 11)$.	**7**	**L3**	**CO4**
	c	Find least square solution of Ax=b for $A = \begin{bmatrix} 1 & 3 & 5 \\ 1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 3 & 3 \end{bmatrix}, b = \begin{bmatrix} 3 \\ 5 \\ 7 \\ -3 \end{bmatrix}$ by QR-factorization.	**7**	**L3**	**CO4**
Module – 5					
Q.9	**a**	Diagonalize the matrix $A = \begin{bmatrix} 5 & 3 \\ 1 & 3 \end{bmatrix}$ and hence find A^8.	**6**	**L2**	**CO5**
	b	Find the singular value decomposition of the matrix $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$.	**7**	**L3**	**CO5**
	c	Find the minimum and maximum values of $Q(x) = 2x^2 + 2y^2 + z^2$ subject to the constraint $X^T X = I$.	**7**	**L3**	**CO5**
OR					
Q.10	**a**	Orthogonally diagonalize the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$	**6**	**L2**	**CO5**
	b	Find the singular value decomposition of the matrix $A = \begin{bmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{bmatrix}$.	**7**	**L3**	**CO5**
	c	Make the change of variable $X = PY$ that transforms the quadratic form $x_1^2 + 10x_1^2x_2^2 + x_2^2$.	**7**	**L3**	**CO5**

Model Question Paper Set-2 with effect from 2024(CBCS Scheme)

USN

Fourth Semester B.E Degree Examination

Linear Algebra

BCS404D

TIME:03Hours **Max.Marks:100**

Note:

1. Answer any **FIVE** full questions, choosing at least **ONE** question from each **MODULE**
2. M: Marks, L: RBT levels, C: Course outcomes.

		Module - 1	**M**	**L**	**C**
Q.1	**a**	Show that the set V of all polynomials of degree n over a field F is not a vector space over F	**6**	**L2**	**CO1**
	b	The set $W = \{(x, y, z): x - 3y + 4z = 2\}$ of the vector space $R^3(R)$ over the field of Real numbers. Check W is a subspace of $R^3(R)$	**7**	**L2**	**CO1**
	c	Does $\{1 - x + 3x^2,\ 1 + x + 7x^2,\ 1 + 3x + 4x^2\}$ the set of vectors forms a linear independent set.	**7**	**L2**	**CO1**
		OR			
Q.2	**a**	Define a subspace. Prove that the intersection of two subspaces of a vector space V(F) is a subspace of V(F)	**6**	**L2**	**CO1**
	b	Determine whether the following set of vectors is linearly independent or linearly dependent. If the set is linearly dependent, express one vector in the set as a linear combination of the others for the vectors $(1, 0, -1, 0), (1, 2, 3, 4), (-1, -2, 0, 1), (-2, -2, 7, 11)$	**7**	**L2**	**CO1**
	c	Find the value of k do the set of vectors $v_1 = (k, 1,1)$, $v_2 = (0, 1,1)$, $v_3 = (k, 0, k)$ form a basis of $R^3(R)$?	**7**	**L3**	**CO1**
		Module – 2			
Q.3	**a**	Prove that $T: R^3 \rightarrow R^3$be defined by $T(a, b, c) = (3a, a - b, 2a + b + c)$ is a linear transformation.	**6**	**L2**	**CO2**
	b	Verify the Rank- nullity theorem for the T:$R^3 \rightarrow R^3$ defined by T (x, y, z)= $(x + 2y - z, y + z, x + y - 2z)$	**7**	**L2**	**CO2**
	c	Define Singular and non Singular linear transformation. If $T: P_3(R) \rightarrow P_3(R)$ is a Linear Transformation given by $T(p(x)) = p(x + 1) - p(x - 1)$ then Check T is a singular linear transformation or not.	**7**	**L2**	**CO2**
		OR			
Q.4	**a**	Let T: V → W be a linear transformation. Then prove that R(T) is a subspace of W.	**6**	**L2**	**CO2**
	b	Let $T: R^3 \rightarrow R^3$ be a linear transformation defined by	**7**	**L2**	**CO2**

		$T(x,y,z) = (x+3y-2z, 2x+3y, y-z)$. Check whether T is isomorphism and hence find T^{-1}.			
	c	Find the matrix of the linear transformation $T: V_2(R) \rightarrow V_3(R)$ defined by $T(x,y) = (x+y, x, 3x-y)$ with respect to $B_1 = \{(1,1),\ (3,1)\}$, $B_2 = \{(1,1,1), (1,1,1), (1,0,0)\}$	7	L3	CO2
		Module – 3			
Q.5	a	Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ $and\ f(x) = x^2 - 4x + 7$. Show that $f(A) = 0$. Use the result to find A^5.	6	L2	CO3
	b	Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ 2 & 1 & 4 \end{bmatrix}$. Hence Compute$A^{-1}$.	7	L2	CO3
	c	Find the Eigen values and Eigen values of the matrix $A = \begin{bmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix}$	7	L3	CO3
		OR			
Q.6	a	Find all the Jordan canonical form of A having $C_A(x) = (x-3)^4(x-5)^5$ and $m_A(x) = (x-3)^2(x-5)^2$.	6	L3	CO3
	b	Find the characteristic and minimal polynomials for the matrix $A = \begin{bmatrix} -2 & -6 & -9 \\ 3 & 7 & 9 \\ -1 & -2 & -2 \end{bmatrix}$	7	L3	CO3
	c	Find the least square solution of $AX = B$ for $A = \begin{bmatrix} 1 & -6 \\ 1 & -2 \\ 1 & 1 \\ 1 & 7 \end{bmatrix}$ & $B = \begin{bmatrix} -1 \\ 2 \\ 1 \\ 6 \end{bmatrix}$	7	L3	CO3
		Module – 4			
Q.7	a	Define an inner product space. If V is an inner product space, then for any vectors α, β in V, Prove that $\lVert \alpha + \beta \rVert \leq \lVert \alpha \rVert + \lVert \beta \rVert$.	6	L2	CO4
	b	Apply the Gram-Schmidt orthogonalization process to find an orthonormal basis for the subspace of R 4 spanned by the vectors $v_1 = (1,1,1,1), v_2 = (1,2,4,5), v_3 = (1,-3,-4,-2)$.	7	L3	CO4
	c	Let V be the vector space of all 2 × 3 matrices over R. The matrices A= $\begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & -5 & 2 \\ 1 & 0 & -4 \end{bmatrix}$ Find $i)\ \langle A,B \rangle$, $\langle A,C \rangle$, $\langle B,C \rangle$ $ii)\ \langle 2A+3B, 4C \rangle$ $iii)\ \lVert A \rVert\ and\ \lVert B \rVert$	7	L2	CO4
		OR			
Q.8	a	Prove that every finite dimensional inner product space has an orthonormal basis.	6	L2	CO4
	b	Find an orthonormal basis for the vector space $V_3(R)$ by applying the Gram-Schmidt orthogonalization process to the vectors $(3,0,4), (-1,0,7)\ and\ (2,9,11)$.	7	L3	CO4
	c	Find the QR Factorization of $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$	7	L3	CO4

<table>
<tr><th colspan="6">Module – 5</th></tr>
<tr><td rowspan="3">Q.9</td><td>a</td><td>Convert the quadratic form $Q(x) = x^2 - 8xy - 5y^2$ into quadratic form with no cross-product form.</td><td>6</td><td>L2</td><td>CO5</td></tr>
<tr><td>b</td><td>Find the singular value decomposition of $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$.</td><td>7</td><td>L3</td><td>CO5</td></tr>
<tr><td>c</td><td>Diagonalize the matrix A, given that A= $\begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ Hence find A^4 .</td><td>7</td><td>L2</td><td>CO5</td></tr>
<tr><th colspan="6">OR</th></tr>
<tr><td rowspan="3">Q.10</td><td>a</td><td>Find the Singular value Decomposition of $A = \begin{bmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{bmatrix}$.</td><td>6</td><td>L3</td><td>CO5</td></tr>
<tr><td>b</td><td>Using PCA, Reduce the dimension for the following data
<table>
<tr><td>Feature</td><td>Exp-1</td><td>Exp-2</td><td>Exp-3</td><td>Exp-4</td></tr>
<tr><td>X1</td><td>4</td><td>8</td><td>13</td><td>7</td></tr>
<tr><td>X2</td><td>11</td><td>4</td><td>5</td><td>14</td></tr>
</table></td><td>7</td><td>L2</td><td>CO5</td></tr>
<tr><td>c</td><td>Diagonalize the symmetric matrix $A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$.</td><td>7</td><td>L3</td><td>CO5</td></tr>
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