Model Question Paper-II with effect from 2022 (CBCS Scheme)

USN | | | | | | | | | | |

Second Semester B.E Degree Examination
Mathematics-II for Mechanical Engineering stream-BMATM201

TIME: 03 Hours **Max. Marks: 100**

Note:
1. Answer any **FIVE** full questions, choosing at least **ONE** question from each **module**.
2. VTU Formula Hand Book is permitted.
3. M: Marks, L: Bloom's level, C: Course outcomes.

		Module -1	**M**	**L**	**C**
Q.01	a	Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{-x-z}^{x+z}(x+y+z)\ dy\ dx\ dz$	**7**	L3	**CO1**
	b	Evaluate $\int_{0}^{4a}\int_{x}^{2\sqrt{ax}} x^2\ \ dy\ dx$ by changing the order of integration.	**7**	L3	**CO1**
	c	Show that $\gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$	**6**	L2	**CO1**
		OR			
Q.02	a	Evaluate $\int_{1}^{2}\int_{3}^{4}(xy+e^y)\ \ dy\ dx$	**7**	L3	**CO1**
	b	Find by double integration area between the parabolas $y^2=4ax$ and $x^2=4ay$	**7**	L2	**CO1**
	c	Write a modern mathematical tool program to find the volume of the tetrahedron bounded by the planes $x=0, y=0\ and\ \frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$	**6**	L3	**CO5**
		Module-2			
Q. 03	a	Find the angle between the surfaces $x^2+y^2-z^2=4$ and $z=x^2+y^2-13$ at $(2,1,2)$	**7**	L2	**CO2**
	b	If $\vec{F}=\nabla(xy^3z^2)$ find $div\ \vec{F}$ and $curl\ \vec{F}$ at the point $(1,-1,\ 1)$	**7**	L2	**CO2**
	c	Define a solenoidal vector. Find the value of a for which $\vec{F}=(x+3y)\hat{\imath}+(y-2z)\hat{\jmath}+(x+az)\hat{k}$ is solenoidal.	**6**	L2	**CO2**
		OR			
Q.04	a	Using Green's theorem, evaluate $\int_C (xy+y^2)dx+x^2dy$, where C is the closed curve of the region bounded by $y=x$ and $y=x^2$.	**7**	L3	**CO2**

	b	Apply Stoke's theorem to evaluate $\int_c \vec{F} \cdot \overrightarrow{dr}$, where $\vec{F} = y^2\hat{\imath} + x^2\hat{\jmath} - (x+z)\hat{k}$ and C is the boundary of the triangle with the vertices (0, 0, 0), (1, 0, 0), (1, 1, 0).	**7**	L3	**CO2**
	c	Write the modern mathematical tool program to find the divergence of the vector field $\vec{F} = x^2yz\hat{\imath} + y^2zx\hat{\jmath} + z^2xy\hat{k}$	**6**	L3	**CO5**
Module-3					
Q. 05	a	Form the PDE by eliminating the arbitrary function from $f(x+y+z, x^2+y^2+z^2) = 0$,	**7**	L2	**CO3**
	b	Solve $\frac{\partial^2 z}{\partial x^2} = xy$, subject to the conditions that $\frac{\partial z}{\partial x} = \log(1+y)$ *when* $x = 1$ and $z = 0$ *when* $x = 0$	**7**	L3	**CO3**
	c	Derive one-dimensional wave equation.	**6**	L2	**CO3**
OR					
Q. 06	a	Form the PDE by eliminating the arbitrary constants a *and* b from $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$	**7**	L2	**CO3**
	b	Solve $\frac{\partial^2 z}{\partial y^2} = z$, given that $y = 0, z = e^x, \frac{\partial z}{\partial y} = e^{-x}$	**7**	L3	**CO3**
	c	Solve $x^2(y^2 - z^2)p + y^2(z^2 - x^2)q = z^2(x^2 - y^2)$ using Lagrange's multipliers.	**6**	L3	**CO3**
Module-4					
Q. 07	a	Find an approximate value of the root of the equation $xe^x - 2 = 0$, in the interval using the Regula-Falsi method.	**7**	L3	**CO4**
	b	Using Newton's divided difference formula, evaluate $f(4)$ from the following table: <table><tr><td>x</td><td>0</td><td>2</td><td>3</td><td>6</td></tr><tr><td>$f(x)$</td><td>-4</td><td>2</td><td>14</td><td>158</td></tr></table>	**7**	L3	**CO4**
	c	Evaluate $\int_0^1 \frac{1}{1+x^2} dx$ by taking 7 ordinates using the Trapezoidal rule.	**6**	L3	**CO4**
OR					
Q. 08	a	Using the Newton-Raphson method, find the real root of the equation $xsinx + \cos x = 0$, which is nearer to $x = \pi$, correct to three decimal places.	**7**	L3	**CO4**

<table>
<tr><td></td><td>b</td><td>Using Newton's forward interpolation formula, find y at $x = 5$
<table><tr><td>x</td><td>4</td><td>6</td><td>8</td><td>10</td></tr><tr><td>y</td><td>1</td><td>3</td><td>8</td><td>16</td></tr></table></td><td>7</td><td>L3</td><td>CO4</td></tr>
<tr><td></td><td>c</td><td>Evaluate $\int_0^5 \frac{1}{4x+5} dx$ by using Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule by taking 7 ordinates. Hence find the value of $log\ 5$.</td><td>6</td><td>L3</td><td>CO4</td></tr>
<tr><td colspan="6" align="center">Module-5</td></tr>
<tr><td>Q. 09</td><td>a</td><td>Find by Taylor's series method the value of y at $x = 0.1$ to 5 places of decimals from $\frac{dy}{dx} = xy^2 - 1, y(0) = 1$.</td><td>7</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>b</td><td>Using the Runge-Kutta method of fourth order, find y(0.2) given that $\frac{dy}{dx} = \frac{y-x}{y+x}, y(0) = 1$, taking $h = 0.2$</td><td>7</td><td>L3</td><td>CO4</td></tr>
<tr><td></td><td>c</td><td>Given that $\frac{dy}{dx} = x - y^2$ and the data $y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795$, $y(0.6) = 0.1762$. Compute y at x = 0.8 by applying Milne's method.</td><td>6</td><td>L3</td><td>CO4</td></tr>
<tr><td colspan="6" align="center">OR</td></tr>
<tr><td>Q. 10</td><td>a</td><td>Using the modified Euler's method, find y(0.1) given that $\frac{dy}{dx} = x^2 + y$ and $y(0) = 1$ take step $h = 0.05$ and perform two modifications in each stage.</td><td>7</td><td>L3</td><td>CO4</td></tr>
<tr><td></td><td>b</td><td>Using the Runge-Kutta method of fourth order find $y(0.1)$ given that $\frac{dy}{dx} = x^2 + y,\ y(0) = -1$.</td><td>7</td><td>L3</td><td>CO4</td></tr>
<tr><td></td><td>c</td><td>Using modern mathematical tools write a code to find $y(0.1)$, given $\frac{dy}{dx} = x - y, y(0) = 1$ by Taylor's Series.</td><td>6</td><td>L3</td><td>CO5</td></tr>
</table>

<table>
<tr><td rowspan="4">Bloom's Taxonomy Levels</td><td colspan="3" align="center">Lower-order thinking skills</td></tr>
<tr><td>Remembering (knowledge): L_1</td><td>Understanding (Comprehension): L_2</td><td>Applying (Application): L_3</td></tr>
<tr><td colspan="3" align="center">Higher-order thinking skills</td></tr>
<tr><td>Analyzing (Analysis): L_4</td><td>Valuating (Evaluation): L_5</td><td>Creating (Synthesis): L_6</td></tr>
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Note:
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2. VTU Formula Hand Book is permitted.
3. M: Marks, L: Bloom's level, C: Course outcomes.

		Module -1	**M**	**L**	**C**
Q.01	a	Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2+y^2)dy\,dx$	**7**	L3	**CO1**
	b	Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyzdz\,dy\,dx$	**7**	L3	**CO1**
	c	Prove that $\int_0^{\frac{\pi}{2}} \sqrt{\sin\theta}\,d\theta \times \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{\sin\theta}} = \pi$	**6**	L2	**CO1**
		OR			
Q.02	a	Evaluate $\int_0^{\infty} \int_x^{\infty} \frac{1}{ye^y} dy\,dx$ by changing the order of integration	**7**	L3	**CO1**
	b	By changing into polar coordinates, evaluate $$\int_0^a \int_0^{\sqrt{a^2-x^2}} (x^2+y^2)\,dxdy$$	**7**	L3	**CO1**
	c	Using modern mathematical tools write a program to evaluate $\int_{-c}^{c} \int_{-b}^{b} \int_{-a}^{a} (x^2+y^2+z^2)dz\,dy\,dx$	**6**	L3	**CO5**
		Module-2			
Q. 03	a	If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $\lvert\vec{r}\rvert = r$, find $grad\left(div\,\frac{\vec{r}}{r}\right)$	**7**	L2	**CO2**
	b	Find the constants a, b and c such that the vector $\vec{F} = (x+y+az)\hat{\imath} + (x+cy+2z)\hat{\jmath} + (bx+2y-z)\hat{\jmath} + (x+cy+2z)\hat{k}$ is irrotational.	**7**	L2	**CO2**
	c	Find the directional derivative of x^2yz^3 at $(1,1,1)$ in the direction of $\hat{\imath} + \hat{\jmath} + 2\hat{k}$	**6**	L2	**CO2**
		OR			
Q.04	a	Find the work done by a force $\vec{f} = (2y-x^2)\hat{\imath} + 6yz\hat{\jmath} - 8xz^2\hat{k}$ from the point $(0,0,0)$ to the point $(1,1,1)$ along the straight–line joining these points.	**7**	L2	**CO2**

	b	Evaluate $\int_c [xydx + xy^2dy]$ by Green's theorem where c is the square in the xy plane with vertices $(1,0), (-1,0), (0,1)$ and $(0,-1)$	**7**	L3	**CO2**
	c	Using modern mathematical tools write a program to evaluate $\int_c [(y - \sin x)dx + \cos x\, dy]$, where c is the plane triangle enclosed by the lines $y = 0$, $x = \frac{\pi}{2}$ and $y = \frac{2x}{\pi}$, Using Green's theorem.	**6**	L3	**CO5**
		Module-3			
Q. 05	a	Form the partial differential equation by eliminating the arbitrary function f from $z = y^2 + 2f\left(\frac{1}{x} + log\, y\right)$	**7**	L2	**CO3**
	b	Solve $\frac{\partial^2 z}{\partial x^2} = a^2 z$ given that $x = 0, z = 0$ and $\frac{\partial z}{\partial x} = a \sin y$	**7**	L3	**CO3**
	c	Solve $(x + 2z)p + (4zx - y)q = (2x^2 + y)$	**6**	L3	**CO3**
		OR			
Q. 06	a	Form the partial differential equation by eliminating the arbitrary constants a *and* b from $(x - a)^2 + (y - b)^2 + z^2 = c^2$	**7**	L2	**CO3**
	b	Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ for which $\frac{\partial z}{\partial y} = -2 \sin y$, when $x = 0$ and $z = 0$ if y is an odd multiple of $\frac{\pi}{2}$.	**7**	L3	**CO3**
	c	Derive one-dimensional heat equation in the standard form as $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$	**6**	L2	**CO3**
		Module-4			
Q. 07	a	Compute the real root of the equation $x \log_{10} x - 1.2 = 0$ lies between 2 and 3 by the Regula-Falsi method. Carry out four approximations.	**7**	L2	**CO4**
	b	The area A of a circle corresponds to the diameter (D) is given below: <table><tr><td>D</td><td>80</td><td>85</td><td>90</td><td>95</td><td>100</td></tr><tr><td>A</td><td>5026</td><td>5674</td><td>6362</td><td>7088</td><td>7854</td></tr></table> Find the area corresponding to the diameter 105 by using the appropriate interpolation formula	**7**	L3	**CO4**
	c	Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{Sin\theta}\, d\theta$ by taking 7 ordinates using Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule.	**6**	L3	**CO4**
		OR			
Q. 08	a	Find the real root of the equation $xe^x = 2$ correct to three decimal places using the Newton-Raphson method.	**7**	L3	**CO4**

<table>
<tr><td></td><td>b</td><td>From the data given in the following table, find the number of students who obtained marks between 40 and 45:
<table>
<tr><td>Marks</td><td>30 - 40</td><td>40 - 50</td><td>50 - 60</td><td>60 - 70</td><td>70 - 80</td></tr>
<tr><td>No. of students</td><td>31</td><td>42</td><td>51</td><td>35</td><td>31</td></tr>
</table></td><td>7</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>c</td><td>Compute the value of y when $x = 3$ using Lagrange's interpolation formulae given
<table>
<tr><td>x</td><td>-2</td><td>-1</td><td>1</td><td>2</td></tr>
<tr><td>y</td><td>-7</td><td>2</td><td>0</td><td>11</td></tr>
</table></td><td>6</td><td>L3</td><td>CO4</td></tr>
<tr><td colspan="6">Module-5</td></tr>
<tr><td>Q. 09</td><td>a</td><td>Solve the differential equation $\frac{dy}{dx} = x^2 + y^2$ given $y(0) = 1$ to find the value of $y(0.1)$ by using the Taylor series method taking the terms up to 4^{th} order</td><td>7</td><td>L3</td><td>CO4</td></tr>
<tr><td></td><td>b</td><td>Apply the Runge-Kutta method of fourth order, to find an approximate value of y at $x = 0.1$, given that $\frac{dy}{dx} = 3e^x + 2y$ with $y(0) = 0$ and $h = 0.1$.</td><td>7</td><td>L3</td><td>CO4</td></tr>
<tr><td></td><td>c</td><td>Given $y' = x^2 + \frac{y}{2}$ and $y(1) = 2, y(1.1) = 2.2156, y(1.2) = 2.4649,$ $y(1.3) = 2.7514$, find $y(1.4)$ using Milne's predictor and corrector formulae</td><td>6</td><td>L3</td><td>CO4</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>Q. 10</td><td>a</td><td>Use modified Euler's method to find $y(0.2)$ given
$\frac{dy}{dx} = 3x + \frac{y}{2}, y(0) = 1$ taking $h = 0.1$
(perform two iterations at each step)</td><td>7</td><td>L3</td><td>CO4</td></tr>
<tr><td></td><td>b</td><td>Use the Runge-Kutta method of 4^{th} order to solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1$ at $x = 0.2$</td><td>7</td><td>L3</td><td>CO4</td></tr>
<tr><td></td><td>c</td><td>Using modern mathematical tools write a program to find y when $x = 0.8$, given $\frac{dy}{dx} = x - y^2$ $y(0) = 0, y(0.2) = 0.2000,$ $y(0.4) = 0.0795, y(0.6) = 0.1762$, Using Milne's predictor-corrector method.</td><td>6</td><td>L3</td><td>CO5</td></tr>
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