Model Question Paper-II with effect from 2022 (CBCS Scheme)

USN | | | | | | | | | | |

Second Semester B.E Degree Examination
Mathematics-II for Computer Science Engineering-BMATS201

TIME: 03 Hours **Max. Marks: 100**

Note:
1. Answer any **FIVE** full questions, choosing at least **ONE** question from each **module**.
2. VTU Formula Hand Book is permitted.
3. M: Marks, L: Bloom's level, C: Course outcomes.

		Module -1	**M**	**L**	**C**
Q.01	a	Evaluate $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2+y^2+z^2)\ dx\,dy\,dz$	**7**	L2	**CO1**
	b	Evaluate $\int_0^1\int_x^{\sqrt{x}} xy\ dy\,dx$ by changing the order of integration.	**7**	L3	**CO1**
	c	Show that $\gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$	**6**	L2	**CO1**
		OR			
Q.02	a	Evaluate $\int_0^1\int_0^{\sqrt{1-y^2}}(x^2+y^2)dxdy$ by changing into polar coordinates	**7**	L3	**CO1**
	b	Find by double integration, the area between the parabolas $y^2=4ax$ and $x^2=4ay$.	**7**	L2	**CO1**
	c	Using Mathematical tools, write the code to find the area of an ellipse by double integration $A=4\int_0^a\int_0^{\frac{b}{a}\sqrt{a^2-x^2}} dydx$.	**6**	L3	**CO5**
		Module-2			
Q. 03	a	Find the directional derivative of $\phi=x^2\,yz+4xz^2$ at (1, -2, -1) in the direction of the vector $2\hat{\imath}-\hat{\jmath}-2\hat{k}$.	**7**	L2	**CO2**
	b	If $\vec{F}=\nabla(xy^3z^2)$, find $div\,\vec{F}$ and $curl\,\vec{F}$ at the point $(1,-1,\ 1)$	**7**	L2	**CO2**
	c	Prove that the spherical coordinate system is orthogonal.	**6**	L2	**CO2**
		OR			
Q.04	a	Find the angle between the surfaces $x^2+y^2-z^2=4$ and $z=x^2+y^2-13$ at (2,1, 2)	**7**	L3	**CO2**
	b	Show that the vector $\vec{F}=\frac{x\hat{\imath}+y\hat{\jmath}}{x^2+y^2}$ is both solenoidal and irrotational.	**7**	L2	**CO2**
	c	Using Mathematical tools, write the code to find the curl of	**6**	L3	**CO5**

		$\vec{F} = xy^2\hat{\imath} + 2x^2yz\,\hat{\jmath} - 3yz^2\hat{k}$			
Module-3					
Q. 05	a	Let $V = R^3$ be a vector space and consider the subset W of V consisting of vectors of the form (a, a^2, b), where the second component is the square of the first. Is W a subspace of V?	**7**	L2	**CO3**
	b	Find the basis and the dimension of the subspace spanned by the vectors $\{(2, 4, 2), (1, -1, 0), (1, 2, 1), (0, 3, 1)\}$ in $V_3(R)$.	**7**	L2	**CO3**
	c	Find the kernel and range of the linear operator $T(x, y, z) = (x + y,\ z)$ of $R^3 \to R^2$	**6**	L2	**CO3**
OR					
Q. 06	a	Let $f(x) = 2x^2 - 5$ and $g(x) = x + 1$. Show that the function $h(x) = 4x^2 + 3x - 7$ lies in the subspace Span {f, g} of P_2.	**7**	L2	**CO3**
	b	Prove that the transformation $T; R^2 \to R^2$ defined by $T(x, y) = (3x,\ x + y)$ is linear. Find the images of the vectors (1, 3) and (–1, 2) under this transformation.	**7**	L2	**CO3**
	c	Show that the functions f(x) = 3x – 2 and g(x) = x are orthogonal in P_n with inner product $\langle f, g\rangle = \int_0^1 f(x)g(x)dx$.	**6**	L2	**CO3**
Module-4					
Q. 07	a	Find an approximate value of the root of the equation $xe^x = 3$, using the Regula-Falsi method, carry out three iterations.	**7**	L2	**CO4**
	b	Using Newton's divided difference formula, evaluate $f(8)$ from the following <table><tr><td>x</td><td>4</td><td>5</td><td>7</td><td>10</td><td>11</td><td>13</td></tr><tr><td>$f(x)$</td><td>48</td><td>100</td><td>294</td><td>900</td><td>1210</td><td>2028</td></tr></table>	**7**	L2	**CO4**
	c	Evaluate $\int_0^1 \frac{1}{1+x^2}dx$ using the Trapezoidal rule by taking 6 divisions.	**6**	L3	**CO4**
OR					
Q. 08	a	Find the real root of the equation $\cos x = xe^x$, which is nearer to $x = 0.5$ by the Newton-Raphson method, correct to three decimal places.	**7**	L2	**CO4**
	b	Given, $\sin 45^0 = 0.7071, \sin 50^0 = 0.7660, \sin 55^0 = 0.8192,$	**7**	L2	**CO4**

		$\sin 60^0 = 0.8660$, find $\sin 48^0$ using Newton's forward interpolation formula.			
	c	Evaluate $\int_0^3 \frac{1}{4x+5} dx$ by using Simpson's 1/3rd rule by taking 7 ordinates.	**6**	L3	**CO4**
		Module-5			
Q. 09	a	By Taylor's series method, find the value of y at $x = 0.1$ and $x = 0.2$ to 5 places of decimals from $\frac{dy}{dx} = x^2y - 1, y(0) = 1$.	**7**	L2	**CO4**
	b	Using the Runge-Kutta method of fourth order, find $y(0.1)$ given that $\frac{dy}{dx} = 3e^x + 2y,\ y(0) = 0$, taking $h = 0.1$	**7**	L2	**CO4**
	c	Given that $\frac{dy}{dx} = x - y^2$ and the data $y(0) = 0,\ y(0.2) = 0.02,\ y(0.4) = 0.0795,\ y(0.6) = 0.1762$ compute y at $x = 0.8$ by applying Milne's method.	**6**	L2	**CO4**
		OR			
Q. 10	a	Using the modified Euler's method, find $y(0.1)$ given that $\frac{dy}{dx} = x^2 + y$ and $y(0) = 1$ take step $h = 0.05$ and perform two modifications in each stage.	**7**	L2	**CO4**
	b	Using the Runge-Kutta method of fourth order, find $y(0.2)$ given that $\frac{dy}{dx} = \frac{y-x}{y+x},\ y(0.1) = 1.0912$, taking $h = 0.1$	**7**	L2	**CO4**
	c	Using Mathematical tools, write the code to find the solution of $\frac{dy}{dx} = 1 + \frac{y}{x}$ at $y(2)$ taking $h = 0.2$. Given that $y(1) = 2$ by Runge-Kutta 4th order method.	**6**	L3	**CO5**

Model Question Paper-I with effect from 2022 (CBCS Scheme)

USN | | | | | | | | | | |

Second Semester B.E Degree Examination
Mathematics-II for Computer Science Engineering-BMATS201

TIME: 03 Hours **Max. Marks: 100**

Note:
1. Answer any **FIVE** full questions, choosing at least **ONE** question from each **module**.
2. VTU Formula Hand Book is permitted.
3. M: Marks, L: Bloom's level, C: Course outcomes.

		Module -1	**M**	**L**	**C**
Q.01	a	Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} \; dz\, dy\, dx$	**7**	L2	**CO1**
	b	Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} \; dx\, dy$ by changing into polar coordinates.	**7**	L2	**CO1**
	c	Show that $\beta(m,n) = \frac{\gamma(m)\gamma(n)}{\gamma(m+n)}$	**6**	L2	**CO1**
		OR			
Q.02	a	Evaluate $\int_0^a \int_y^a \frac{x}{x^2+y^2} dxdy$ by changing the order of integration	**7**	L2	**CO1**
	b	Using double integration find the area of a plate in the form of a quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	**7**	L3	**CO1**
	c	Write the codes to find the volume of the tetrahedron bounded by the planes $x = 0,\; y = 0 \; and \; z = 0, \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ using Mathematical tools.	**6**	L3	**CO5**
		Module-2			
Q. 03	a	Find $\nabla\phi$, if $\phi = x^3 + y^3 + z^3 - 3xyz$ at the point $(1, -1, 2)$	**7**	L2	**CO2**
	b	If $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$ find $div\, \vec{F}$ and $curl\, \vec{F}$	**7**	L2	**CO2**
	c	Express the vector $\vec{A} = z\hat{\imath} - 2x\hat{\jmath} + y\hat{k}$ in cylindrical coordinates.	**6**	L2	**CO2**
		OR			
Q.04	a	Find the directional derivative of $\phi = x^2\, yz + 4xz^2$ at (1, -2, -1) in the direction of the vector $2\hat{\imath} - \hat{\jmath} - 2\hat{k}$.	**7**	L2	**CO2**
	b	Show that the spherical coordinate system is orthogonal	**7**	L3	**CO2**
	c	Using the Mathematical tools, write the codes to find the gradient of	**6**	L3	**CO5**

		$\emptyset = x^2yz$.			
Module-3					
Q. 05	a	Prove that the subset W = {(x, y, z) | x – 3y + 4z = 0} of the vector space R^3 is a subspace of R^3.	**7**	L3	**CO3**
	b	Determine whether the matrix $\begin{bmatrix} -1 & 7 \\ 8 & -1 \end{bmatrix}$ is a linear combination of $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 2 & -3 \\ 0 & 2 \end{bmatrix}$ and $\begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$ in the vector space M_{22} of 2×2 matrices.	**7**	L2	**CO3**
	c	Find the matrix of the linear transformation T: $V_2(R) \to V_3(R)$ such that T (–1, 1) = (–1, 0, 2) and T (2, 1) = (1, 2, 1).	**6**	L2	**CO3**
OR					
Q. 06	a	Show that the set S = {(1, 2, 4), (1, 0, 0), (0, 1, 0), (0, 0, 1)} is linearly dependent.	**7**	L2	**CO3**
	b	Let P_n be the vector space of real polynomial functions of degree ≤ n. Show that the transformation $T: P_2 \to P_1$ defined by $T(ax^2 + bx + c) = (a + b)x + c$ is linear.	**7**	L2	**CO3**
	c	Verify the Rank-nullity theorem for the linear transformation T: $V_3(R) \to V_2(R)$ defined by $T(x, y, z) = (y - x,\ y - z)$.	**6**	L2	**CO3**
Module-4					
Q. 07	a	Find the real root of the equation $3x = \cos x + 1$ correct to three decimal places using Newton's Raphson method.	**7**	L2	**CO4**
	b	Find y at $x = 5$ if $y(1) = -3,\ y(3) = 9, y(4) = 30, y(6) = 132$ using Lagrange's interpolation formula.	**7**	L2	**CO4**
	c	Evaluate $\int_0^1 \frac{1}{1+x} dx$ by taking 7 ordinates and by using Simpson's 3/8 rule.	**6**	L3	**CO4**
OR					
Q. 08	a	Find an approximate root of the equation $x^3 - 3x + 4 = 0$ using the method of false position, correct to three decimal places which lie between -3 and -2. (Carry out three iterations).	**7**	L2	**CO4**

	b	Using Newton's appropriate interpolation formula, find the values of y at $x = 8$ and at $x = 22$ from the following table: x: 0, 5, 10, 15, 20, 25 y: 7, 11, 14, 18, 24, 32	**7**	L3	**CO4**
	c	Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\cos x}\, dx$ by using the Trapezoidal rule by taking 7 ordinates.	**6**	L2	**CO4**
Module-5					
Q. 09	a	Solve $y'(x) = 3x + \frac{y}{2}$, $y(0) = 1$ then find $y(0.2)$ with $h = 0.2$ using modified Euler's method.	**7**	L2	**CO4**
	b	Apply Runge-Kutta method of fourth order to find an approximate value of y when $x = 0.2$ given that $\frac{dy}{dx} = x + y$ and $y = 1$ when $x = 0$.	**7**	L2	**CO4**
	c	Given $\frac{dy}{dx} = xy + y^2$, $y(0) = 1, y(0.1) = 1.1169$, $y(0.2) = 1.2773$, $y(0.3) = 1.5049$ compute $y(0.4)$ using Milne's method.	**6**	L2	**CO4**
OR					
Q. 10	a	Employ Taylor's series method to obtain approx. value of y at $x = 0.2$ for the differential equation $\frac{dy}{dx} = 2y + 3e^x, y(0) = 0$.	**7**	L2	**CO4**
	b	Using the Runge-Kutta method of fourth order, solve $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$ with $y(0) = 1$ at $x = 0.2$.	**7**	L2	**CO4**
	c	Write the Mathematical tool codes to solve the differential equation $\frac{dy}{dx} - 2y = 3e^x$ with $y(0) = 0$ using the Taylors series method at $x = 0.1(0.1)0.3$.	**6**	L3	**CO5**