Fourth Semester B.E.(CBCS) Examination
Engineering Mathematics-IV
(Common to all Branches)

Time: 3 Hrs  Max.Marks: 80

Note: Answer any FIVE full questions, choosing at least ONE question from each module.
Use of statistical tables allowed.

Module-I

1. (a) Solve \( \frac{dy}{dx} = x^2 y^2 + 1, y(0) = 1 \) using Taylor’s series method considering up to fourth degree terms and, find the \( y(0.1) \) \hfill (05 Marks)

(b) Use Runge - Kutta method of fourth order to solve \( 10 \frac{dy}{dx} = x^2 + y^2, y(0) = 1 \) to find \( y(0.2) \).
(Take \( h = 0.2 \)) \hfill (05 Marks)

(c) Given that \( \frac{dy}{dx} = x(1 + y^2) \) and \( y(1) = 2, y(2.1) = 1.2330, y(2.2) = 1.5480, \) \& \( y(2.3) = 1.9790 \)
find \( y(1.4) \), using Adam-Bashforth predictor-corrector method. \hfill (06 Marks)

OR

2. (a) Solve the differential equation \( \frac{dy}{dx} = -xy^2 \) under the initial condition \( y(0) = 2 \) by using modified Euler’s method at the point \( x = 0.1 \). Perform three iterations at each step, taking \( h = 0.05 \). \hfill (05 Marks)

(b) Use fourth order Runge - Kutta method, to find \( y(0.2) \), given \( \frac{dy}{dx} = 3x + y, y(0) = 1 \). \hfill (05 Marks)

(c) Apply Milne’s predictor-corrector formulae to compute \( y(1.2) \) given

\[
\frac{dy}{dx} = 3x - 4y^2 \quad \text{with} \quad \begin{array}{c|c|c|c|c}
 x & 0 & 0.3 & 0.6 & 0.9 \\
 y & 1.0 & 1.3020 & 1.3795 & 1.4762 \\
\end{array}
\] \hfill (06 Marks)

Module-II

3. (a) By Runge - Kutta method, solve \( \frac{d^2y}{dx^2} = x \left( \frac{dy}{dx} \right)^2 - y^2 \) for \( x = 0.2 \), correct to four decimal places, using initial conditions \( y(0) = 1, y'(0) = 0 \). \hfill (05 Marks)

(b) If \( \alpha \) and \( \beta \) are two distinct roots of \( J_n(x) = 0 \), then prove that \( \int_0^1 x J_n(\alpha x) J_n(\beta x) \, dx = 0 \) if \( \alpha \neq \beta \). \hfill (05 Marks)

(c) Express \( f(x) = x^3 - 5x^2 + 14x + 5 \) in terms of Legendre polynomials. \hfill (06 Marks)

OR
4. (a) Apply Milne’s predictor-corrector method to compute \( y(0.4) \) given the differential equation \( \frac{d^2y}{dx^2} + \frac{dy}{dx} = 2e^x \) and the following table of initial values:

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>2</td>
<td>2.01</td>
<td>2.04</td>
<td>2.09</td>
</tr>
<tr>
<td>( y' )</td>
<td>0</td>
<td>0.20</td>
<td>0.40</td>
<td>0.60</td>
</tr>
</tbody>
</table>

(05 Marks)

(b) With usual notation, show that \( J_{-1/2}(x) = \sqrt{(2/\pi x)} \cos x \).

(05 Marks)

(c) With usual notation, derive the Rodrigues’s formula viz., \( P_n(x) = \frac{1}{2^n n!} d^n \left( x^2 - 1 \right)^n \).

(06 Marks)

Module-III

5. (a) Derive Cauchy-Riemann equation in cartesian form.

(05 Marks)

(b) evaluate \( \int \frac{\sin \frac{\pi z}{2} + \cos \frac{\pi z}{2}}{z^2} \frac{dz}{(z-1)(z-2)} \) where \( C \) is the circle \( |z| = 3 \), using Cauchy’s residue theorem.

(05 Marks)

(c) Discuss the transformation \( w = z^2 \).

(06 Marks)

OR

6. (a) Find the analytic function whose real part is \( r^2 \cos 2\theta \).

(05 Marks)

(b) State and prove Cauchy’s theorem.

(05 Marks)

(c) Find the bilinear transformation which maps the points \( z = \infty, i, 0 \) into the points \( w = -1, -i, 1 \).

(06 Marks)

Module-IV

7. (a) Derive mean and variance of the Poisson distribution.

(05 Marks)

(b) A random variable \( X \) has the following probability function for various values of \( x \):

<table>
<thead>
<tr>
<th>( X(= x_i) )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(x) )</td>
<td>0</td>
<td>( k )</td>
<td>( 2k )</td>
<td>( 2k )</td>
<td>( 3k )</td>
<td>( k^2 )</td>
<td>( 2k^2 )</td>
<td>( 7k^2 + k )</td>
</tr>
</tbody>
</table>

Find (i) the value of \( k \) (ii) \( P(x < 6) \) (iii) \( P(x \geq 6) \)

(05 Marks)

(c) Let \( X \) be the random variable with the following distribution and \( Y \) is defined by \( X^2 \):

<table>
<thead>
<tr>
<th>( X(= x_i) )</th>
<th>-2</th>
<th>-1</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x_i) )</td>
<td>1/4</td>
<td>1/4</td>
<td>1/4</td>
<td>1/4</td>
</tr>
</tbody>
</table>

Determine (i) the distribution of \( g \) of \( Y \) (ii) joint distribution of \( X \) and \( Y \) (iii) \( E(X), E(Y), E(XY) \).

(06 Marks)
8. (a) When a coin is tossed 4 times find, using binomial distribution, the probability of getting (i) exactly one head (ii) at most 3 heads (iii) at least 3 heads. (05 Marks)

(b) In a normal distribution, 31% of the items are under 45 and 8% of the items are over 64%. Find the mean and standard deviation of the distribution. (05 Marks)

(c) A fair coin is tossed thrice. The random variables \( X \) and \( Y \) are defined as follows:
\[ X = 0 \text{ or } 1 \text{ according as head or tail occurs on the first}; \]
\[ Y = \text{Number of heads}. \]
Determine (i) the distribution of \( X \) and \( Y \) (ii) joint distribution of \( X \) and \( Y \). (06 Marks)

**Module-V**

9. (a) Define the terms: (i) Null hypothesis (ii) Confidence intervals (iii) Type-I and Type-II errors (05 marks) 

(b) Ten individuals are chosen at random from a population and their heights in inches are found to be 63, 63, 66, 67, 68, 69, 70, 71, 71. Test the hypothesis that the mean height of the universe is 66 inches. (\( t_{0.05} = 2.262 \) for 9 d.f.). (05 marks) 

(c) Show that probability matrix 
\[ P = \begin{bmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{bmatrix} \]
is regular stochastic matrix and find the associated unique fixed probability vector. (06 marks)