

VISVESVARAYATECHNOLOGICALUNIVERSITY

BELAGAVI



MATHEMATICS HANDBOOK

III and IV Semester BE Program

(Common to all Lateral Entry MATDIP Students)

(FOR SYLLABUS SCHEMES 2002/2006/2010/2015/2017/2018)





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Complex Number

A number of the form $a + ib$, where a and b are real numbers, is called a complex number, a is called the real part and b is called the imaginary part of the complex number.

Equality of complex numbers $a + ib = c + id \Leftrightarrow a = c \text{ and } b = d$

Addition of complex numbers $(a + ib) + (c + id) = (a + c) + i(b + d)$

Subtraction of complex numbers $(a + ib) - (c + id) = (a - c) + i(b - d)$

Multiplication of complex numbers $(a + ib) \cdot (c + id) = (ac - bd) + i(ad + bc)$

Division of complex numbers $\frac{a+ib}{c+id} = \frac{a+ib}{c+id} \cdot \frac{c-id}{c-id} = \frac{ac+bd}{c^2+d^2} + i \frac{bc-ad}{c^2+d^2}$

Polar form of complex numbers $a + ib = r(\cos\theta + i\sin\theta)$

For any non-zero complex number $z = a + ib$ ($a \neq 0, b \neq 0$), there exists

the complex number $\frac{a}{a^2+b^2} + i \frac{-b}{a^2+b^2}$, denoted by $\frac{1}{z}$ or z^{-1} called multiplicative inverse

of z such that $(a + ib) \left(\frac{a}{a^2+b^2} + i \frac{-b}{a^2+b^2} \right) = 1 + i0 = 1$

For any integer $k, i^{4k} = 1, i^{4k+1} = i, i^{4k+2} = -1, i^{4k+3} = -i$.

The conjugate of the complex number $z = a + ib$, denoted by $\bar{z}$, is given by

$$\bar{z} = a - ib.$$





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The plane having a complex number assigned to each of its point is called the complex plane or the Argand plane.

The Argand plane, the $|x + iy| = \sqrt{x^2 + y^2}$.

n^{th} Derivatives of Standard Functions:

$$D^n[(ax + b)^m] = m(m-1)(m-2) \dots (m-n+1)(ax + b)^{m-n} \cdot a^n.$$

$$D^n[(ax + b)^n] = n! a^n$$

$$D^n[x^n] = n!$$

$$D^n\left[\frac{1}{ax + b}\right] = \frac{(-1)^n n! a^n}{(ax + b)^{n+1}}$$

$$D^n[\log(ax + b)] = \frac{(-1)^{n-1} (n-1)! a^n}{(ax + b)^n}$$

$$D^n[a^{mx}] = a^{mx} (m \log a)^n$$

$$D^n[e^{ax}] = a^n e^{ax}$$

$$D^n[\sin(ax + b)] = a^n \sin\left(ax + b + \frac{n\pi}{2}\right)$$

$$D^n[\cos(ax + b)] = a^n \cos\left(ax + b + \frac{n\pi}{2}\right)$$





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$$D^n[e^{ax}\sin(ax+b)] = (a^2 + b^2)^{n/2}e^{ax}\sin\left(bx + c + n\tan^{-1}\left(\frac{b}{a}\right)\right)$$

$$D^n[e^{ax}\cos(ax+b)] = (a^2 + b^2)^{n/2}e^{ax}\cos\left(bx + c + n\tan^{-1}\left(\frac{b}{a}\right)\right)$$

Polar Coordinates and Polar Curves:

Angle between Polar vector and Tangent

$$\tan\phi = r \frac{d\theta}{dr} \text{ or } \cot\phi = \frac{1}{r} \frac{dr}{d\theta}$$



Angle of Intersection of the Curves

$$|\phi_1 - \phi_2| = \tan^{-1} \left\{ \left| \frac{\tan\phi_1 - \tan\phi_2}{1 + \tan\phi_1 \cdot \tan\phi_2} \right| \right\}$$

Orthogonal Condition

$$|\phi_1 - \phi_2| = \frac{\pi}{2} \text{ or } \tan\phi_1 \cdot \tan\phi_2 = -1$$

Series Expansion:

Taylor's Series expansion about the point $x = a$

$$y(x) = y(a) + \frac{(x-a)^1}{1!} y'(a) + \frac{(x-a)^2}{2!} y''(a) + \frac{(x-a)^3}{3!} y'''(a) + \dots$$





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Maclaurian's Series expansion about the point $x = 0$

$$y(x) = y(0) + \frac{x^1}{1!} y'(0) + \frac{x^2}{2!} y''(0) + \frac{x^3}{3!} y'''(0) + \frac{x^4}{4!} y''''(0) + \dots$$

Euler's theorem on Homogeneous Function and Corollary

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = nu$$

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = n \frac{f(u)}{f'(u)}$$

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xyy^2 \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = n(n-1)u$$



Composite Function

If $z = f(x, y)$ and $x = \phi(t)$, $y = \psi(t)$ then $\frac{dz}{dt} = \frac{\partial z}{\partial x} \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt}$

If $z = f(x, y)$ and $x = \phi(u, v)$, $y = \psi(u, v)$ then

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial u} \text{ and } \frac{\partial z}{\partial v} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial v} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial v}$$

If $u = f(r, s, t)$ and $r = g(x, y, z)$, $s = h(x, y, z)$, $t = i(x, y, z)$ then

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r} \frac{\partial r}{\partial x} + \frac{\partial u}{\partial s} \frac{\partial s}{\partial x} + \frac{\partial u}{\partial t} \frac{\partial t}{\partial x},$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial r} \frac{\partial r}{\partial y} + \frac{\partial u}{\partial s} \frac{\partial s}{\partial y} + \frac{\partial u}{\partial t} \frac{\partial t}{\partial y},$$

$$\frac{\partial u}{\partial z} = \frac{\partial u}{\partial r} \frac{\partial r}{\partial z} + \frac{\partial u}{\partial s} \frac{\partial s}{\partial z} + \frac{\partial u}{\partial t} \frac{\partial t}{\partial z}.$$





Jacobians $J = \frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$ and $\frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix}$



Reduction Formulae

$$\int_0^{\frac{\pi}{2}} \cos^n x dx = \int_0^{\frac{\pi}{2}} \sin^n x dx = \begin{cases} \frac{(n-1)(n-3)(n-5) \dots 1}{n(n-2)(n-4) \dots 2} \frac{\pi}{2} & \text{when } n \text{ is even} \\ \frac{(n-1)(n-3)(n-5) \dots 3}{n(n-2)(n-4) \dots 1} & \text{when } n \text{ is odd} \end{cases}$$

$$\int_0^{\frac{\pi}{2}} \sin^m x \cos^n x dx = \begin{cases} \frac{(m-1)(m-3) \dots (n-1)(n-3) \dots \pi}{(m+n)(m+n-2)(m+n-4) \dots 2} \frac{\pi}{2} & \text{when } m \text{ \& } n \text{ is even} \\ \frac{(m-1)(m-3) \dots (n-1)(n-3) \dots}{(m+n)(m+n-2)(m+n-4) \dots} & \text{otherwise} \end{cases}$$

Multiple Integrals

Area $A = \iint_A dx dy$ - Cartesian form

Area $A = \iint_A r dr d\theta$ - Polar form

Volume $V = \iiint_A dx dy dz$ - Cartesian form

Volume $V = \iint_A z dx dy$ – by double integral

Gamma Function: $\Gamma(n) = \int_0^{\infty} e^{-x} x^{n-1} dx = 2 \int_0^{\infty} e^{-t^2} t^{2n-1} dt$





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Beta Function: $\beta(m, n) = \int_0^1 x^{n-1} (1-x)^{m-1} dx = 2 \int_0^{\frac{\pi}{2}} \sin^{2n-1} \theta \cos^{2m-1} \theta d\theta$

Beta and Gamma relation $\beta(m, n) = \frac{\Gamma(m) \Gamma(n)}{\Gamma(m+n)}$ and $\Gamma(1/2) = \sqrt{\pi}$.

Vector Calculus

Position Vector $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Magnitude $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$



Dot Product of unit vectors $\hat{i} \cdot \hat{i} + \hat{j} \cdot \hat{j} + \hat{k} \cdot \hat{k} = 1$ and $\hat{i} \cdot \hat{j} + \hat{j} \cdot \hat{k} + \hat{k} \cdot \hat{i} = 0$

Cross Product of unit vectors $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$ and $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{i} = \hat{j}$.

Angle between two vectors $\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|}$; $\sin \theta = \frac{|\vec{A} \times \vec{B}|}{|\vec{A}| |\vec{B}|}$

Unit vector $\hat{A} = \frac{\vec{A}}{|\vec{A}|}$

Velocity $\vec{V} = \frac{ds}{dt}$

Acceleration $\vec{a} = \frac{d^2s}{dt^2}$

For any vectors $\vec{A} = (a_1\hat{i} + b_1\hat{j} + c_1\hat{k})$, $\vec{B} = (a_2\hat{i} + b_2\hat{j} + c_2\hat{k})$ &

$\vec{C} = (a_3\hat{i} + b_3\hat{j} + c_3\hat{k})$





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Dot product of two vectors $\vec{A} \cdot \vec{B} = a_1a_2 + b_1b_2 + c_1c_2$

Cross product of two vectors $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

Scalar triple product $\vec{A} \cdot (\vec{B} \times \vec{C}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$



Vector Calculus

Velocity $\vec{v}(t) = \frac{d\vec{r}}{dt}$

Acceleration $\vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$

The unit tangent vector $\hat{T} = \frac{\frac{d\vec{r}}{dt}}{\left| \frac{d\vec{r}}{dt} \right|}$

Angle between the tangents $\cos\theta = \frac{\vec{T}_1 \cdot \vec{T}_2}{|\vec{T}_1||\vec{T}_2|}$

Component of velocity $C.V = \vec{v} \cdot \hat{n}$, where $\hat{n}$ is the unit vector

Component of accelerations $C.A = \vec{a} \cdot \hat{n}$

Tangent component of acceleration $T.C.A = \vec{a} \cdot \vec{v} / |\vec{v}|$

Normal Component of acceleration:





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$$N.C.A = \left| \vec{a} - (\text{tangential component}) \times \left(\frac{\vec{v}}{|\vec{v}|} \right) \right|$$

$$\text{Gradient of } \phi: \text{grad}\phi = \nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$$

$$\text{Unit vector normal to the surface: } \hat{n} = \frac{\nabla\phi}{|\nabla\phi|}$$

$$\text{Directional derivative along vector } \vec{n} \text{ is } D.D = \nabla\phi \cdot \hat{n}$$

$$\text{Angle between the surfaces } \cos\theta = \frac{\nabla\phi_1 \cdot \nabla\phi_2}{|\nabla\phi_1||\nabla\phi_2|}$$

$$\text{Divergence of vector field } \vec{F} = f_1\hat{i} + f_2\hat{j} + f_3\hat{k}$$

$$\text{div } \vec{F} = \nabla \cdot \vec{F} = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$$

$$\text{Curl of vector field } \vec{F}: \text{curl } \vec{F} = \nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_1 & f_2 & f_3 \end{vmatrix}$$

$$\text{Solenoidal vector field: } \text{div } \vec{F} = \nabla \cdot \vec{F} = 0$$

$$\text{Irrotational vector field: } \text{curl } \vec{F} = \nabla \times \vec{F} = 0$$

List of vector identities





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$$\text{curl}(\text{grad}\phi) = \nabla \times \nabla\phi = 0$$

$$\text{div}(\text{curl}\vec{F}) = \nabla \cdot (\nabla \times \vec{F}) = 0$$

$$\text{div}(\phi\vec{F}) = \phi(\text{div}\vec{F}) + \text{grad}\phi \cdot \vec{F}$$

$$\text{curl}(\phi\vec{F}) = \phi(\text{curl}\vec{F}) + \text{grad}\phi \times \vec{F}$$



Linear Algebra

Inverse of square matrix A : $A^{-1} = \frac{(\text{adj}A)}{|A|}$

Rank of a matrix A : The number of non-zero rows in the echelon form of A is equal to rank of A .

Normal form of a matrix: $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$

Gauss Elimination method

The system is reduced to upper triangular system from which the unknowns are found by back substitutions.

Eigenvalues

Roots of the characteristic equation $|A - \lambda I| = 0$.

Eigen Vectors





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Non-zero solution $x = x_i$ of $|A - \lambda I|x = 0$.

Diagonal form

$$D = P^{-1}AP = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$



Nature, Rank and Index of Quadratic forms:

- **Positive-definite** if all the Eigen values of A are positive.
- **Positive-semi definite** if all the Eigen values of A are non-negative and at least one of the eigenvalues is zero.
- **Negative-definite** if all the Eigen values of A are negative.
- **Negative-semi definite** if all the Eigen values of A are negative and at least one of the Eigen values is zero.
- **Indefinite** if the matrix A has both positive and negative eigenvalues.
- **Rank** the number of non-zero terms.
- **Index** the number of positive terms.
- **Signature** the number of positive terms minus the number of negative terms.

