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18AI56

Fifth Semester B.E. Degree Examination, Feb./Mar. 2022
Mathematics for Machine Learning

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Define vector spaces, subspaces. Show that the set S = {(1, 0, 1), (1, 1, 0), (–1, 0, –1) is linearly dependant in $V_3(R)$. **(10 Marks)**

b. By Gaussian elimination, find the inverse of the matrix :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}.$$ **(10 Marks)**

OR

2 a. Solve the system of linear equations using elementary row operations.

$$2x + y + 4z = 12$$
$$4x + 11y - z = 33$$
$$8x - 3y + 2z = 20.$$ **(06 Marks)**

b. Define :
i) Norm on vector space U
ii) Inner product in R^n
iii) Angle between vectors. **(06 Marks)**

c. Find rank of the matrix A by reducing into echelon form

$$A = \begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}.$$ **(08 Marks)**

Module-2

3 a. Define :
i) Orthogonal sets
ii) Projection vector. **(04 Marks)**

b. If $y = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$ and $u = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$ find the orthogonal projection of y onto u and the orthogonal set. **(06 Marks)**

c. Diagonalize the matrix $A = \begin{bmatrix} 6 & -2 & -1 \\ -2 & 6 & -1 \\ -1 & -1 & 5 \end{bmatrix}$. **(10 Marks)**

OR

4 a. Find all the eigen values and corresponding eigen vectors of the matrix :

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}.$$ **(10 Marks)**

b. Find the singular value decomposition of $A = \begin{bmatrix} 2 & 2 & -2 \\ 2 & 2 & -2 \\ -2 & -2 & 6 \end{bmatrix}$. **(10 Marks)**

Module-3

5 a. Expand $f(x) = \sin x + \cos x$ by Maclaurin's series up to terms containing x^4. **(06 Marks)**

b. Compute the derivative of $f(x) = x^n$, $n \in N$ using the definition of derivative as limit. **(06 Marks)**

c. If $f(x) = \sqrt{x^2 + \exp(x^2)} + \cos(x^2 + \exp(x^2))$ find $\frac{df}{dx}$ using the following computation graph and intermediate variables a, b, c, d, $a = x^2$, $b = \exp(a)$, $c = a + b$, $d = \sqrt{c}$, $e = \cos(c)$, $f = d + e$.

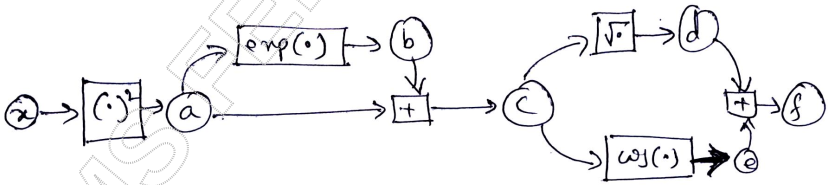

Fig.Q5(c) **(08 Marks)**

OR

6 a. If $f(x_1, x_2) = x_1^2 x_2 + x_1 x_2^3 \in R$, then find the gradient of f. **(06 Marks)**

b. Define some identities, for computing gradients. **(06 Marks)**

c. If $u = f(p, q, r)$ where $p = \frac{x}{y}, q = \frac{y}{z}, r = \frac{z}{x}$ find $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}$. **(08 Marks)**

Module-4

7 a. A bag contains 8 white and 6 red balls. Find the probability of drawing two balls of the same colour. **(06 Marks)**

b. State and prove Baye's theorem of conditional probability. **(06 Marks)**

c. A random variable x has the following probability distribution :

x	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

i) Find k

ii) Mean and variance of the distribution

iii) $P(x < 3)$, $P(4 \le x < 7)$. **(08 Marks)**

OR

8 a. A pair of dice is tossed twice. Find the probability of scoring 7 points i) once ii) twice. **(06 Marks)**

b. A continuous random variable X has a p.d.f $f(x) = K\,x^2 e^{-x}$, $x \geq 0$. Find K, mean and variance. **(06 Marks)**

c. Find :
i) Marginal distribution f(x) and g(y)
ii) E(X), E(Y), E(XY) for the following joint distribution of X and Y.

X \ Y	–4	2	7
1	⅛	¼	⅛
5	¼	⅛	⅛

(08 Marks)

Module-5

9 a. Find the maximum of Z = 2x + 3y
subject to the constraints $x + y \leq 30$,
$y \geq 30$,
$0 \leq y \geq 12$,
$x - y \geq 0$
and $0 \leq x \leq 20$. **(10 Marks)**

b. For convex functions f(y) and g(x), show that

$$\min_x f(Ax) + g(x) = \min_u - f^*(u) - g^*(-A^T u).$$ **(10 Marks)**

OR

10 a. Consider the linear program given below and derive the dual linear program using Lagrage duality.

$$\min_{x \in R^2} \frac{1}{2}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^T \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 5 \\ 3 \end{bmatrix}^T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Subject to $\begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \leq \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$. **(10 Marks)**

b. Discuss the optimization using Gradient descent, conjugate gradient, subgradient methods. Differential the methods if any. **(10 Marks)**

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