CBCS SCHEME

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18CB36

Third Semester B.Tech. Degree Examination, Feb./Mar. 2022
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Define the following terms with an example:
(i) Reflexive relation (ii) Symmetric relation (iii) Transitive relation
(iv) Equivalence relation (v) Partial ordering relation (vi) Relation on set A **(06 Marks)**

b. A certain computer centre employs 100 programmers. Of these 47 can program in FORTRAN, 35 in PASCAL, 20 in COBOL, 23 in FORTRAN and PASCAL, 12 in COBOL and FORTRAN, 11 in PASCAL and COBOL, and 5 in all the three of these. How many can program in none of these languages? **(07 Marks)**

c. Find the eigen values and the eigen vectors of the matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$. **(07 Marks)**

OR

2 a. If A, B, C are finite sets, prove that
$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$ **(06 Marks)**

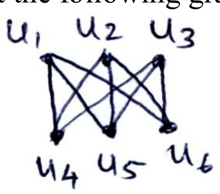
b. Let A = {1, 2, 3, 4, 6} and R be a relation on A defined by xRy if and only if "x is a multiple of y". Represent R as a set of ordered pairs. **(07 Marks)**

c. Let $A \subset Z$ and R be a relation on A defined as xRy if and only if $x - y$ is a multiple of 5. Verify R is an equivalence relation. **(07 Marks)**

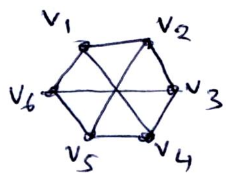
Module-2

3 a. For the universe of all integers, let
p(x) : x > 0, q(x) : x is even
r(x) : x is a perfect square
s(x) : x is divisible by 3
t(x): x is divisible by 7
Write down the following quantified statements in symbolic form:
(i) Atleast one integer is even.
(ii) There exists a positive integer that is even.
(iii) Some even integers are divisible by 3
(iv) Every integer is either even or odd
(v) If x is even, then x is not divisible by 3.
(vi) There exists a perfect square that is divisible by 7. **(06 Marks)**

b. Show that the following graphs are isomorphic.



G_1



G_2 **(07 Marks)**

c. Using truth table, prove the following logical equivalence:
$[\, p \rightarrow (q \rightarrow r)\,] \Leftrightarrow [(p \wedge q) \rightarrow r]$ **(07 Marks)**

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OR

4 a. Test the validity of the following argument:

I will become famous, or I will not become a musician.
I will become a musician.

$\therefore$ I will become famous. **(06 Marks)**

b. Give direct proof of the statement:
"For all integers k and ℓ, if k and ℓ are both odd, then $k + \ell$ is even and $k\ell$ is odd. **(07 Marks)**

c. Define the following terms:
(i) Planar Graph (ii) Hamilton graph
(iii) Chromatic number (iv) Complete graph **(04 Marks)**

d. Find the chromatic number of the following graph [Fig.Q4(d)].

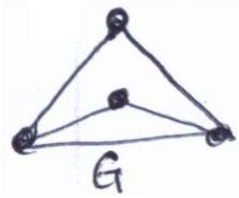

Fig.Q4(d) **(03 Marks)**

Module-3

5 a. Define the following terms with an example:
(i) Length (ii) Concatenation (iii) Replication
(iv) Reversal (v) Prefix **(10 Marks)**

b. Convert the following NFA to DFA and describe the language it accepts.
$M = (\{P, Q, R, S, T\}, \{0, 1\}, \delta, P, \{S, T\})$ and δ is given as

	0	1
P	{P, Q}	{P}
Q	{R, S}	{T}
R	{P, R}	{T}
S	-	-
T	-	-

(10 Marks)

OR

6 a. If w and x are strings then prove that $(wx)^R = x^R w^R$. **(10 Marks)**

b. Convert the following $\in$-NFA to DFA.

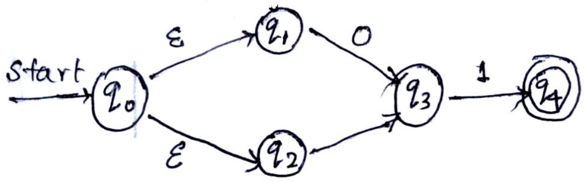

Fig.Q6(b) **(10 Marks)**

Module-4

7 a. Show that regular languages are closed under complement and intersection. **(10 Marks)**

b. Show that $L = \{ww \mid w \in \{a, b\}^*\}$ is not regular. **(10 Marks)**

OR

8 a. Write the applications of regular expressions. **(10 Marks)**

b. Obtain an NFA for the regular expression $a^* + b^* + c^*$. **(10 Marks)**

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Module-5

9 a. Define ambiguity. Consider the following grammer:

$E \rightarrow E + E \mid E - E$

$E \rightarrow E*E \mid E/E \mid (E) \mid id$

Show that the grammer is ambiguous. **(10 Marks)**

b. Obtain a PDA to accept $L = \{a^n b^n \mid n \geq 0\}$. **(10 Marks)**

OR

10 a. Write the general procedure to convert CFG to PDA. **(10 Marks)**

b. Convert the following grammer into equivalent PDA.

$E \rightarrow E + T$

$E \rightarrow T$

$T \rightarrow T*F$

$T \rightarrow F$

$F \rightarrow (E)$

$F \rightarrow id$ **(10 Marks)**

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