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Fourth Semester B.E. Degree Examination, July/August 2021
Engineering Statistics and Linear Algebra

Time: 3 hrs.

Max. Marks: 100

Note: 1. Answer any FIVE full questions.**2. Use of normalized Gaussian Random variable table is permitted.**

- 1 a. Define a random variable and discuss the following terms associated with random variables:
- Sample space
 - Probability Mass function
 - Probability density function
 - Cumulative distribution function.
- (05 Marks)
- b. Given the data in the following table:

K	Y_K	$P(Y_K)$
1	2.1	0.20
2	3.2	0.21
3	4.8	0.19
4	5.4	0.14
5	6.9	0.26

- Plot the Pdf and Cdf of the discrete random variable Y.
 - Compute the mean and variance of Y.
- (08 Marks)
- c. If 'X' is a random variable uniformly distributed in the interval [a b], obtain the expression for mean, variance and mean square value.
- (07 Marks)
- 2 a. The Pdf for a random variable Y is $f_y(Y) = 1.5(1 - y^2)$ $0 < y < 1$ what are mean, mean square, variance of Y.
- (08 Marks)
- b. It is given that $E[X] = 2.0$ and that $E[X^2] = 6$.
- Find the standard deviation of X
 - If $Y = 6X^2 + 2X - 13$ find μ_Y
- (06 Marks)
- c. The normalized Gaussian random variable is given as
- $$f_x(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \quad -\alpha < x < \alpha$$
- Obtain the characteristic function for this random variable.
- (06 Marks)
- 3 a. A bivariate Pdf is given as
- $$f_{XY}(x, y) = 0.2\delta(x) \delta(y) + 0.3\delta(x-1) \delta(y) + 0.3\delta(x) \delta(y-1) + C\delta(x-1) \delta(y-1)$$
- What is the value of the constant C?
 - What are the Pdfs for X and Y?
 - What is $F_{XY}(x, y)$ when $(0 < x < 1)$ and $(0 < y < 1)$?
 - What are $F_{XY}(x, \alpha)$ and $F_{XY}(\alpha, y)$
 - Are X and Y independent?
- (08 Marks)
- b. The mean and variance of random variable X are -1 and 2. The mean and variance of random variable Y are 3 and 4. The correlation coefficient $\rho_{XY} = 0.5$. What are the covariance $COV[XY]$ and the correlation $E[XY]$.
- (05 Marks)
- c. Write a short note on Chi-square random variable and students random variable.
- (07 Marks)

- 4 a. X is a random variable, $\mu_X = 4$ and $\sigma_X = 5$, Y is a random variable, $\mu_Y = 6$ and $\sigma_Y = 7$. The correlation coefficient is 0.2. If $U = 3X + 2Y$. What are $\text{var}[u]$, $\text{cov}[uX]$ and $\text{cov}[uY]$? (08 Marks)
- b. Let 'X' and 'Y' be exponentially distributed random variable with $f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$
- c. Obtain the characteristic function and Pdf of $W = X + Y$. (06 Marks)
- The Random variables X_i have same mean of $m_x = 4$ and variance of $\sigma_x^2 = 1.5$. For $w = \sum_{i=1}^{150} X_i$, determine m_w and σ_w^2 . Also for $w = \frac{1}{150} \sum_{i=1}^{150} X_i$, determine m_y and σ_y^2 . Comment on the result. (06 Marks)
- 5 a. Briefly explain the following terms:
i) Random process
ii) Stationary process
iii) Ergodic process. (04 Marks)
- b. Discuss Auto correlation and Auto covariance function of a random process. Also mention properties of Auto correlation function. (06 Marks)
- c. A random process is described by $X(t) = A \cos(w_c t + \theta)$ where A, w_c are constants and ' θ ' is a random variable uniformly distributed between $\pm\pi$. Is $X(t)$ wide sense stationary? If so, then what are the mean and the auto correlation function for the random process? (10 Marks)
- 6 a. Let $X(t)$ and $Y(t)$ be two independent jointly wide sense stationary random process defined as $X(t) = A \cos(w_1 t + \theta_1)$, $Y(t) = B \cos(w_2 t + \theta_2)$. Here θ_1 and θ_2 are dependent random variables distributed uniformly between $-\pi$ to π obtain the auto correlation function of the multiplication process. $w(t) = Y(t) X(t)$. (06 Marks)
- b. Establish a relationship between power spectrum and Auto-correlation function. (06 Marks)
- c. The power spectral density (PsD) of a wide sense stationary random process is given as
- $$S_X(w) = \begin{cases} 2 \cos\left(\frac{\pi w}{2w_m}\right) & \text{for } -w_m < w \leq w_m \\ 0 & \text{otherwise} \end{cases}$$
- The Psd of a carrier random process is given as $s_c(w) = 10\pi[\delta(w - w_0) + \delta(w + w_0)]$ where $w_0 \gg w_m$. Obtain the Psd of the modulated signal. (08 Marks)
- 7 a. Write the vector $U = (1, 3, 9)$ as a linear combination of the vectors $u_1 = (2, 1, 3)$, $u_2 = (1, -1, 1)$, $u_3 = (3, 1, 5)$. (05 Marks)
- b. Determine whether or not each of the following forms a basis, $x_1 = (2, 2, 1)$, $x_2 = (1, 3, 7)$, $x_3 = (1, 2, 2)$ in R^3 . (05 Marks)
- c. Determine whether the given transformation is linear or not
 $T: R^2 \rightarrow R^3$
 $T(x, y) = [x-y, x+y, 2x]$ (10 Marks)
- 8 a. What is a vector space? Define the four fundamental vector spaces. (05 Marks)
- b. Apply Gramschmidt process to the vectors $v_1 = (2, 2, 1)$, $v_2 = (1, 3, 1)$, $v_3 = (1, 2, 2)$ to obtain an orthonormal basis for $v_3(R)$ with standard inner product. (10 Marks)
- c. Let w be the subspace of R^5 spaced by $x_1 = (1, 2, -1, 3, 4)$, $x_2 = (2, 4, -2, 6, 8)$, $x_3 = (1, 3, 2, 2, 6)$, $x_4 = (1, 4, 5, 1, 8)$, $x_5 = (2, 7, 3, 3, 9)$. Find the dimension of w . (05 Marks)

- 9 a. If a 4×4 matrix has $\det A = \frac{1}{2}$, find: i) $\det (2A)$ ii) $\det (-A)$ iii) $\det (A^2)$ iv) $\det (A^{-1})$.

(04 Marks)

- b. By applying row operations, produce an upper triangular matrix and hence compute determinant of the matrix.

$$B = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & -2 \end{bmatrix}$$

(06 Marks)

- c. Find a matrix P which diagonalizes a matrix $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$. Hence verify that $D = P^{-1}AP$ and compute A^4 .

(10 Marks)

- 10 a. If $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$ find eigen values and eigen vectors for the matrix A. Can the matrix be diagonalized?

(10 Marks)

- b. Compute $A^T A$ and AA^T . Find eigen values and unit eigen vectors for

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}$$

, multiply the three matrices u $\sum V^T$ to recover A.

(10 Marks)

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