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18EC45

Fourth Semester B.E. Degree Examination, July/August 2021 Signals and Systems

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions.

1 a. Determine each of the following:

i) $f(t) = e^{-3(t-1)}\delta(t)$

ii) $f(t) = \int_{-1}^1 \delta(t^2 - 4)dt$

iii) $f(t) = \int_{-\infty}^{\infty} \delta(t^2 - 4)dt$

(06 Marks)

b. Sketch the waveforms for the following signals:

i) $x_1(t) = u(t) - u(t-3)$

ii) $x_2(t) = u(t+2) - 2u(t) + u(t-2)$

iii) $x_3(t) = -u(t+3) + 2u(t+1) - 2u(t-1) + u(t-3)$

iv) $x_4(t) = r(t+1) - r(t) + r(t-2)$

v) $x_5(t) = r(t+2) - r(t+1) - r(t-1) + r(t-2)$

(06 Marks)

c. Find the even and odd parts of i) $x(n) = u(n)$ ii) $x(n) = \alpha^n u(n)$.

(08 Marks)

2 a. A trapezoidal pulse, $x(t)$ is defined by

$$x(t) = \begin{cases} 5-t, & 4 \leq t \leq 5 \\ 1, & -4 \leq t \leq 4 \\ t+5, & -5 \leq t \leq -4 \\ 0, & \text{otherwise} \end{cases}$$

This pulse is applied to a differentiation having the input-output relation:

$y(t) = \frac{dx(t)}{dt}$. Find the energy of the signal $y(t)$.

(06 Marks)

b. Evaluate the following integrals:

i) $\int_{-1}^1 (3t^2 + 1)dt$

ii) $\int_{-\infty}^{\infty} (t^2 + \cos \pi t) \delta(t-1)dt$

iii) $\int_{-\infty}^{\infty} e^{-t} \delta(2t-2)dt$

(06 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8=50, will be treated as malpractice.

- c. i) Sketch $x(n)$ and then plot its even and odd parts $x(n) = 1 + u(n)$.
 ii) For the signal $x(t)$ shown in Fig.Q.2(c), sketch $x_e(t)$ and $x_o(t)$.

(08 Marks)

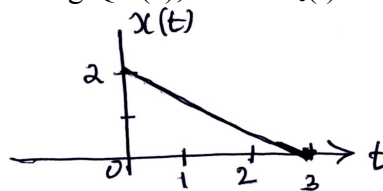


Fig.Q.2(c)

- 3 a. Prove the following:

i) $x_1(n) * h_1(n) = h_1(n) * x_1(n)$

ii) $[x_1(n) * h_1(n)] * h_2(n) = x_1(n) * [h_1(n) * h_2(n)]$

iii) $x_1(n) * [h_1(n) + h_2(n)] = x_1(n) * h_1(n) + x_1(n) * h_2(n)$

(08 Marks)

- b. Prove the following:

i) $x(n) * \delta(n - n_0) = x(n - n_0)$

ii) $\delta(n - \alpha) * \delta(n - \beta) = \delta(n - \alpha - \beta)$

(04 Marks)

- c. Convolute the two continuous-time signals:

i) $x_1(t) = e^{-2t}u(t)$

ii) $x_2(t) = u(t + 2)$

(08 Marks)

- 4 a. An LTI system is characterized by an impulse response, $h(n) = \left(\frac{3}{4}\right)^n u(n)$. Find the step response of the system.

(08 Marks)

- b. Convolute the sequences given below:

i) $x_1(n) = \alpha^n u(n)$

ii) $x_2(n) = \beta^n u(n)$

(08 Marks)

- c. Show that:

$$[x_1(t) * h_1(t)] * h_2(t) = x_1(t) * [h_1(t) * h_2(t)]$$

(04 Marks)

- 5 a. Show that an LTI system is BIBO stable if and only if its impulse response $h(n)$ is absolutely summable. That is, $\sum_{k=-\infty}^{\infty} |h(k)| < \infty$.

(06 Marks)

- b. State and prove the following properties of Fourier series:

- i) Time-shift ii) Time differentiation.

(06 Marks)

- c. Find the Fourier coefficient $X(k)$ for a periodic signal $x(t) = |\sin \pi t|$.

(08 Marks)

- 6 a. Investigate causality and stability of the following systems:

(i) $h(t) = e^{-2|t|}$ (ii) $h(n) = 2^n u(n - 1)$

(06 Marks)

- b. Prove the convolution property of Fourier series.

(04 Marks)

- c. Find the Fourier coefficient $X(k)$ for the periodic signal shown in Fig.Q.6(c). Also, plot magnitude and phase spectra.

(10 Marks)

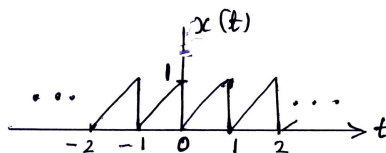


Fig.Q.6(c)

- 7 a. Prove the following properties of Fourier transform:
i) Time-domain convolution ii) Frequency – shift. (06 Marks)
- b. Find the Fourier transform of
- $$x(t) = \begin{cases} 1, & |t| < a \\ 0, & |t| > a \end{cases}$$
- Also, sketch magnitude and phase spectra. (08 Marks)
- c. Find Fourier transform of signum function, $x(t) = \text{sgn}(t)$. (06 Marks)
- 8 a. Prove the following properties of Discrete-Time Fourier Transform (DTFT):
i) Time-shift ii) Time-domain convolution. (06 Marks)
- b. Find the DTFT of $x(n) = a^{|n|}$, $|a| < 1$. Also, sketch magnitude and phase spectra. (08 Marks)
- c. Find inverse DTFT of $X(e^{j\Omega}) = \frac{3 - \frac{1}{4}e^{-j\Omega}}{-\frac{1}{16}e^{-j2\Omega} + 1}$ (06 Marks)
- 9 a. Prove the following properties of Z-transform:
i) Multiplication by 'n' (Ramp) ii) Time-domain convolution. (06 Marks)
- b. Find the inverse Z-transform of $X(z) = \frac{z}{3z^2 - 4z + 1}$ for the ROCs:
i) $|z| > 1$ ii) $|z| < \frac{1}{3}$ iii) $\frac{1}{3} < |z| < 1$ (08 Marks)
- c. Find Z-transform of the sequences given below, using properties
- i) $x(n) = \left(\frac{1}{2}\right)^n u(n) * \left(\frac{1}{3}\right)^n u(n)$
- ii) $x(n) = n u(n)$ (06 Marks)
- 10 a. Find the Z-transform of $x(n) = \sin(\Omega_0 n) u(n)$. (04 Marks)
- b. Find the inverse Z-transform of $X(z) = \frac{z+1}{(z-1)^2 \left(z - \frac{1}{2}\right)}$; ROC: $|z| > 1$. (08 Marks)
- c. A causal LTI system is described by $y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n)$, where $x(n)$ and $y(n)$ are the input and output of the system, respectively.
- i) Find the system function, $H(z)$.
- ii) Find the impulse response, $h(n)$.
- iii) Find the BIBO stability of the system.
- iv) Find the frequency response of the system. (08 Marks)

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