VISVESVARAYA TECHNOLOGICAL UNIVERSITY

BELAGAVI

MATHEMATICS HANDBOOK

I and II Semester BE Program

Effective from the academic year 2022-2023

Derivatives of some standard functions:

$$\frac{d}{dx}(c)=0$$

$$\frac{d}{dx}\left(x^{n}\right)=n\,x^{n-1}$$

$$\frac{d}{dx}\left(e^{x}\right)=e^{x}$$

$$\frac{d}{dx}\left(a^{x}\right)=a^{x}\log a$$

$$\frac{d}{dx}(\sin x)=\cos x$$

$$\frac{d}{dx}(\cos x)=-\sin x$$

$$\frac{d}{dx}(\tan x)=\sec^{2}x$$

$$\frac{d}{dx}(\cot x)=-\cos ec^{2}x$$

$$\frac{d}{dx}(\sec x)=\sec x\tan x$$

$$\frac{d}{dx}(\cos ecx)=-\cos ecx\ \cot x$$

$$\frac{d}{dx}(\log x)=\frac{1}{x}$$

$$\frac{d}{dx}(\log_{a}x)=\frac{\log_{a}e}{x}$$

$$\frac{d}{dx}\left(\sin^{-1}x\right)=\frac{1}{\sqrt{1-x^{2}}}$$

$$\frac{d}{dx}\left(\cos^{-1}x\right)=\frac{-1}{\sqrt{1-x^{2}}}$$

$$\frac{d}{dx}\left(\tan^{-1}x\right)=\frac{1}{1+x^{2}}$$

$$\frac{d}{dx}\left(\cot^{-1}x\right)=\frac{-1}{1+x^{2}}$$

$$\frac{d}{dx}(\sinh x)=\cosh x$$

$$\frac{d}{dx}(\cosh x)=\sinh x$$

$$\frac{d}{dx}(\tanh x)=\sec h^{2}x$$

$$\frac{d}{dx}(\coth x)=-\cos ech^{2}x$$

$$\frac{d}{dx}(\sec hx)=-\sec hx\tanh x$$

$$\frac{d}{dx}(\cos echx)=-\cos echx\coth x$$

Rules of Differentiation:

$$\frac{d}{dx}(cu)=c\frac{du}{dx}$$

$$\frac{d}{dx}(uv)=u\frac{d}{dx}(v)+v\frac{d}{dx}(u)$$

$$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{d}{dx}(u)-u\frac{d}{dx}(v)}{v^{2}}$$

Parametric differentiation:

If $x = x(t)$ & $y = y(t)$ $then$ $\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}$

Chain Rule:

If $y = f(u)$ & $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Integrals of some standard functions:
(Constant of Integration C to be added in all the integrals)

$\int x^n\, dx = \frac{x^{n+1}}{n+1}$

$\int \frac{1}{x} dx = \log x$

$\int \log x dx = x \log x - x, x \neq 0$

$\int k dx = k\, x$

$\int e^x dx = e^x$

$\int a^x dx = \frac{a^x}{\log a}$

$\int \sin x\, dx = -\cos x$

$\int \cos x\, dx = \sin x$

$\int \tan x\, dx = \log(\sec x)$

$\int \cot x\, dx = \log(\sin x)$

$\int \sec x\, dx = \log(\sec x + \tan x)$

$\int \cos ecx\, dx = \log(\cos ecx - \cot x)$

$\int \sec^2 x\, dx = \tan x$

$\int \cos ec^2 x\, dx = -\cot x$

$\int \sec x \tan x\, dx = \sec x$

$\int \cos ecx \;\; \cot x\, dx = -\cos ecx$

$\int \sinh x\, dx = \cosh x$

$\int \cosh x\, dx = \sinh x$

$\int \tanh x\, dx = \log(\cosh x)$

$\int \coth x\, dx = \log(\sinh x)$

$\int \sec h^2 x\, dx = \tanh x$

$\int \cos ech^2 x\, dx = -\coth x$

$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}(x/a)$

$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}(x/a)$

$\int \frac{f'(x)}{f(x)} dx = \log(f(x))$

$\int \frac{f'(x)}{\sqrt{f(x)}} dx = 2\sqrt{f(x)}$

$\int e^{ax} \cos bx\, dx = \frac{e^{ax}}{a^2 + b^2}[a \cos bx + b \sin bx]$

$\int e^{ax} \sin bx\, dx = \frac{e^{ax}}{a^2 + b^2}[a \sin bx - b \cos bx]$

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx \qquad \int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx & \text{if } f(x) \text{ is even function} \\ 0 & \text{if } f(x) \text{ is odd function} \end{cases}$$

$$\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & if \quad f(2a-x) = f(x) \\ 0, & if \quad f(2a-x) = -f(x) \end{cases}$$

Integration by parts:

$$\int u(x)v(x)\,dx = u(x)\left(\int v(x)\,dx\right) - \int \frac{d}{dx}\left(u(x)\right)\left(\int v(x)\,dx\right)dx$$

Bernoulli's rule of integration:

If the 1st function is a polynomial and integration of 2^{nd} function is known. Then

$$\int u(x)v(x)dx = u\int v\,dx - u'\iint v\,dxdx + u''\iiint v\,dxdxdx - u'''\iint\iint v\,dxdxdxdx + \cdots\cdots\cdots$$

Where dashes denote the differentiation of u.

Or

$$\int u(x)v(x)dx = u\cdot v_1 - u'\cdot v_2 + u''\cdot v_3 - u'''\cdot v_4 + \cdots\cdots\cdots$$

Where dashes denote the differentiation of u, v_k denotes the integration of v, k times with respect to *x*.

Vector calculus formulae:

Position vector $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Magnitude $\left|\vec{r}\right| = \sqrt{x^2 + y^2 + z^2}$

Dot product of unit vectors $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$ and $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$

Cross product of unit vectors $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$ and $\hat{i}\times\hat{j} = \hat{k},\ \hat{j}\times\hat{k} = \hat{i},\ \hat{i}\times\hat{k} = -\hat{j}$

Angle between two vectors $\cos\theta = \dfrac{\vec{A}.\,\vec{B}}{\left|\vec{A}\right|\left|\vec{B}\right|}$

Unit vector $\hat{A} = \dfrac{\vec{A}}{\left|\vec{A}\right|}$

Velocity $\vec{V} = \frac{ds}{dt}$

Acceleration $\vec{a} = \frac{d^2 s}{dt^2}$

For any vectors $\vec{A} = (a_1 i + b_1 j + c_1 k)$, $\vec{B} = (a_2 i + b_2 j + c_2 k)$ & $\vec{C} = (a_3 i + b_3 j + c_3 k)$

Dot product of two vectors $\vec{A} \cdot \vec{B} = a_1 a_2 + b_1 b_2 + c_1 c_2$

Cross product of two vectors $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

Scalar triple product $\vec{A} \cdot \left(\vec{B} \times \vec{C} \right) = \left(\vec{A} \times \vec{B} \right) \cdot \vec{C} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

Trigonometric formulae:

- **Identities**

  $\sin^2\theta + \cos^2\theta = 1$

  $1 + \tan^2\theta = \sec^2\theta$

  $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

- **Compound angle formulae**

  $\sin(A+B) = \sin A \cos B + \cos A \sin B$

  $\sin(A-B) = \sin A \cos B - \cos A \sin B$

  $\cos(A+B) = \cos A \cos B - \sin A \sin B$

  $\cos(A-B) = \cos A \cos B + \sin A \sin B$

  $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

  $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

- **Transformation formulae**

  $\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$, $\cos A \sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$

  $\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$, $\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$

  $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$, $\sin C - \sin D = 2\sin\left(\frac{C-D}{2}\right)\cos\left(\frac{C+D}{2}\right)$

  $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$, $\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

- **Multiple angle formulae**

$\sin 2A = 2\sin A\cos A$

$\sin A = 2\sin(A/2)\cos(A/2)$

$\cos 2A = \cos^2 A - \sin^2 A$

$\cos A = \cos^2(A/2) - \sin^2(A/2)$

$\sin^2 A = \dfrac{(1-\cos 2\theta)}{2}$

$\cos^2 A = \dfrac{(1+\cos 2\theta)}{2}$

$\sin^3 A = \dfrac{1}{4}[3\sin A - \sin 3A]$

$\cos^3 A = \dfrac{1}{4}[3\cos A + \cos 3A]$

Hyperbolic and Euler's formulae

$\sinh x = \dfrac{e^x - e^{-x}}{2}$

$\cosh x = \dfrac{e^x + e^{-x}}{2}$

$\cosh^2\theta - \sinh^2\theta = 1$

$e^{i\theta} = \cos\theta + i\sin\theta$

$\sin x = \dfrac{e^{ix} - e^{-ix}}{2i}$

$\cos x = \dfrac{e^{ix} + e^{-ix}}{2}$

Logarithmic formulae:

$\log_e(AB) = \log_e(A) + \log_e(B)$

$\log_e\left(\dfrac{A}{B}\right) = \log_e(A) - \log_e(B)$

$\log_e x^n = n\log_e x$

$\log_a B = \dfrac{\log_e B}{\log_e a}$

$\log_a a = 1$

$\log_a 1 = 0$

$\log_e 0 = -\infty$

Solid geometry formulae:

Distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is $\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

Direction cosines $l = \cos\alpha,\ m = \cos\beta,\ n = \cos\gamma$

Direction ratios $l = \dfrac{a}{\sqrt{a^2+b^2+c^2}},\ m = \dfrac{b}{\sqrt{a^2+b^2+c^2}},\ n = \dfrac{c}{\sqrt{a^2+b^2+c^2}}.$

Direction ratios of a line joining two points $(a, b, c) = (x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1)$

nth Derivatives of standard functions

$$D^n\left[(ax+b)^m\right]=m(m-1)(m-2)......(m-n+1)(ax+b)^{m-n}.a^n$$

$$D^n\left[(ax+b)^n\right]=n!a^n$$

$$D^n\left(x^n\right)=n!$$

$$D^n\left(\frac{1}{ax+b}\right)=\frac{(-1)^n n!a^n}{(ax+b)^{n+1}}$$

$$D^n\left[\log(ax+b)\right]=\frac{(-1)^{n-1}(n-1)!a^n}{(ax+b)^n}.$$

$$D^n\left[a^{mx}\right]=a^{mx}(m\log a)^n$$

$$D^n\left(e^{ax}\right)=a^ne^{ax}$$

$$D^n\left[\sin(ax+b)\right]=a^n\sin(ax+b+n.\pi/2)$$

$$D^n\left[\cos(ax+b)\right]=a^n\cos(ax+b+n.\pi/2)$$

$$D^n\left[e^{ax}\sin(bx+c)\right]=\left(a^2+b^2\right)^{n/2}e^{ax}\sin\left(bx+c+n\tan^{-1}\left(b/a\right)\right)$$

$$D^n\left[e^{ax}\cos(bx+c)\right]=\left(a^2+b^2\right)^{n/2}e^{ax}\cos\left(bx+c+n\tan^{-1}\left(b/a\right)\right)$$

Polar coordinates and polar curves:

Angle between radius vector and tangent

$$\tan\phi=r\frac{d\theta}{dr}\quad\text{or}\quad\cot\phi=\frac{1}{r}\frac{dr}{d\theta}$$

Angle of intersection of the curves

$$|\phi_1-\phi_2|=\tan^{-1}\left\{\left|\frac{\tan\phi_1-\tan\phi_2}{1+\tan\phi_1\tan\phi_2}\right|\right\}$$

Orthogonal condition $|\phi_1-\phi_2|=\frac{\pi}{2}$ or $\tan\phi_1\cdot\tan\phi_2=-1,$

Pedal equation or *p-r* equation

$$P=r\sin\phi$$

$$\frac{1}{p^2}=\frac{1}{r^2}(1+\cot^2\emptyset)=\frac{1}{r^2}\left(1+\frac{1}{r^2}\left(\frac{dr}{d\theta}\right)^2\right)$$

Derivative of arc length:

In Cartesian: $\frac{ds}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ & $\frac{ds}{dy} = \sqrt{1 + \left(\frac{dx}{dy}\right)^2}$

In Polar: $\frac{ds}{dr} = \sqrt{1 + \frac{1}{r^2}\left(\frac{d\theta}{dr}\right)^2}$ & $\frac{ds}{d\theta} = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}$

In Parametric: $\frac{ds}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

$sin\emptyset = r\frac{d\theta}{ds}$ & $cos\emptyset = r\frac{dr}{ds}$

Radius of curvature

In Cartesian form: $\rho = \frac{(1+{y_1}^2)^{\frac{3}{2}}}{y_2}$

In parametric form: $\rho = \frac{(\dot{x}^2+\dot{y}^2)^{\frac{3}{2}}}{\dot{x}\ddot{y}-\dot{y}\ddot{x}}$

In polar from: $\rho = \frac{(r^2+{r_1}^2)^{\frac{3}{2}}}{r^2-rr_1+2{r_1}^2}$

Pedal Equation: $\rho = r\frac{dr}{dp}$

Indeterminate Forms - L'Hospital's rule:

If $f(a) = g(a) = 0$, then $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$

If $f(a) = g(a) = \infty$, then $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$

$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$, $\lim_{n\to 0}(1+n)^{\frac{1}{n}} = e$

$\lim_{\theta\to 0}\frac{sin\theta}{\theta} = 1$, $\lim_{x\to a}\frac{x^n-a^n}{x-a} = na^{n-1}$

Series Expansion:

Taylor's series expansion about the point $x = a$.

$$y(x) = y(a) + \frac{(x-a)}{1!}y'(a) + \frac{(x-a)^2}{2!}y''(a) + \frac{(x-a)^3}{3!}y'''(a) + ..$$

Maclaurin's Series at the point $x = 0$

$$y(x) = y(0) + \frac{x}{1!}y'(0) + \frac{x^2}{2!}y''(0) + \frac{x^3}{3!}y'''(0) + \frac{x^4}{4!}y''''(0) + ...$$

Euler's theorem on homogeneous function and Corollary:

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=n\,u \qquad \text{(Theorem)}$$

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=n\,\frac{f(u)}{f'(u)} \qquad \text{(Corollary 1)}$$

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=n(n-1)u \qquad \text{(Corollary 2)}$$

Composite function:

If $z=f(x,y)$ and $x=\phi(t), y=\psi(t)$ then $\frac{dz}{dt}=\frac{\partial z}{\partial x}\frac{dx}{dt}+\frac{\partial z}{\partial y}\frac{dy}{dt}$

If $z=f(x,y)$ and $x=\phi(u,v), y=\psi(u,v)$

$$\frac{\partial z}{\partial u}=\frac{\partial z}{\partial x}\frac{\partial x}{\partial u}+\frac{\partial z}{\partial y}\frac{\partial y}{\partial u} \quad \& \quad \frac{\partial z}{\partial v}=\frac{\partial z}{\partial x}\frac{\partial x}{\partial v}+\frac{\partial z}{\partial y}\frac{\partial y}{\partial v}$$

If $u=f(r,s,t)$ *and* $r=\phi(x,y,z), s=\psi(x,y,z), t=\xi(x,y,z)$

$$\frac{\partial u}{\partial x}=\frac{\partial u}{\partial r}\frac{\partial r}{\partial x}+\frac{\partial u}{\partial s}\frac{\partial s}{\partial x}+\frac{\partial u}{\partial t}\frac{\partial t}{\partial x}$$

$$\frac{\partial u}{\partial y}=\frac{\partial u}{\partial r}\frac{\partial r}{\partial y}+\frac{\partial u}{\partial s}\frac{\partial s}{\partial y}+\frac{\partial u}{\partial t}\frac{\partial t}{\partial y}$$

$$\frac{\partial u}{\partial z}=\frac{\partial u}{\partial r}\frac{\partial r}{\partial z}+\frac{\partial u}{\partial s}\frac{\partial s}{\partial z}+\frac{\partial u}{\partial t}\frac{\partial t}{\partial z}$$

Jacobians:

$$J=\frac{\partial(u,v)}{\partial(x,y)}=\begin{vmatrix}\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y}\\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y}\end{vmatrix} \quad \text{and} \quad \frac{\partial(u,v,w)}{\partial(x,y,z)}=\begin{vmatrix}\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z}\\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z}\\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z}\end{vmatrix}$$

Multiple Integrals:

Area A= $\iint_A dx\,dy$ - Cartesian form

Area A= $\iint_A r\,dr\,d\theta$ -Polar form

Volume V= $\iiint_V dx\,dydz$ - Cartesian form

Volume V= $\iint_A zdx\,dy$ - by double integral

Gamma function: $\Gamma(n) = \int_0^\infty e^{-x} x^{n-1} dx = 2\int_0^\infty e^{-t^2} t^{2n-1} dt$

Beta function: $\beta(m,n) = \int_0^1 x^{n-1}(1-x)^{m-1}\,dx = 2\int_0^{\frac{\pi}{2}} \sin^{2n-1}\theta\,\cos^{2m-1}\theta\,d\theta$

Beta and Gamma relation: $\beta(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ and $\Gamma\left(\tfrac{1}{2}\right) = \sqrt{\pi}$

Vector Calculus:

Velocity $\vec{v}(t) = \dfrac{d\vec{r}}{dt}$

Acceleration $\vec{a}(t) = \dfrac{d\vec{v}}{dt} = \dfrac{d^2\vec{r}}{dt^2}.$

The unit tangent vector $\hat{T} = \dfrac{d\vec{r}}{dt} \Big/ \left|\dfrac{d\vec{r}}{dt}\right|$

Angle between the tangents $\cos\theta = \dfrac{\vec{T}_1 . \vec{T}_2}{\left|\vec{T}_1\right|\left|\vec{T}_2\right|}$

Component of velocity $C.V = \vec{v}\,.\,\hat{n}$, Where $\hat{n}$ is the unit vector

Component of accelerations $C.A = \vec{a}\,.\,\hat{n}$

Tangential component of acceleration $T.C.A = \vec{a}\,.\,\vec{v}\Big/\left|\vec{v}\right|$

Normal component of acceleration $N.C.A = \left|\vec{a} - (\text{tangential component}) \times \left(\vec{v}\Big/\left|\vec{v}\right|\right)\right|$

Gradient of ϕ

$$\text{grad}\,\phi = \nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial z}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$$

Unit vector normal to the surface

$$\hat{n} = \frac{\nabla\phi}{|\nabla\phi|}$$

Directional Derivative: $\text{D. D} = \nabla\phi\,.\,\hat{n}$

Angle between the surfaces

$$\cos\theta = \frac{\nabla\phi_1 . \nabla\phi_2}{|\nabla\phi_1||\nabla\phi_2|}$$

Divergence of **vector field** $\vec{F} = f_1 i + f_2 j + f_3 k$

$$\nabla.\vec{F} = divF = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial z} + \frac{\partial f_3}{\partial z}$$

Curl of **vector field** $\vec{F}$

$$\nabla \times \vec{F} = curl\,\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_1 & f_2 & f_3 \end{vmatrix}$$

Solenoidal vector field

$$div\,\vec{F} = \nabla \cdot \vec{F} = 0$$

Irrotational vector field

$$Curl\,\vec{F} = \nabla \times \vec{F} = 0.$$

List of vector identities

$$curl(grad\phi) = \nabla \times \nabla\phi = 0.$$

$$div(curl\,\vec{F}) = \nabla \cdot \left(\nabla \times \vec{F}\right) = 0.$$

$$div(\phi\vec{F}) = \phi\left(div\,\vec{F}\right) + grad\phi \cdot \vec{F}$$

$$curl(\phi\vec{F}) = \phi\left(curl\,\vec{F}\right) + grad\phi \times \vec{F}$$

Reduction formulae

$$\int_0^{\pi/2} \cos^n x dx = \int_0^{\pi/2} \sin^n x dx = \begin{cases} \frac{(n-1)(n-3)(n-5).........1}{n(n-2)(n-4)..............2}\frac{\pi}{2} & \text{when n is even} \\ \frac{(n-1)(n-3)(n-5).........3}{n(n-2)(n-4)................1}1 & \text{when n is odd} \end{cases}$$

$$\int_0^{\pi/2} \sin^m x \cos^n x dx = \begin{cases} \frac{(m-1)(m-3).........(n-1)(n-3).........}{(m+n)(m+n-2)(m+n-4).............2}\frac{\pi}{2} & \text{when } m \text{ and } n \text{ is even} \\ \frac{(m-1)(m-3).........(n-1)(n-3).........}{(m+n)(m+n-2)(m+n-4)..............} & \text{other cases} \end{cases}$$

Differential Equations:

Differential Equations	Solution/ substitution
Linear in y $\quad \frac{dy}{dx}+Py=Q$	$y(I.F)=\int Q(I.F)dx+C$
Linear in x $\quad \frac{dx}{dy}+Px=Q$	$x(I.F)=\int Q(I.F.)dy+C$
Bernoulli's $\quad \frac{dy}{dx}+P\,y=Q\,y^n$	divide by y^n and $Put\ y^{1-n}=z$
Exact differential equation $Mdx+Ndy=0$ **with** $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}.$	$\int_{y\text{ constant}} M\,dx+\int(\text{terms of } N \text{ not containing } x)\,dy=C$
Not exact differential equation **Case 1** : If $Mdx+Ndy=0$ and $Mdx+Ndy=0$	$I.F.=\frac{1}{Mx+Ny},\ Mx+Ny\neq 0$
Case 2 : If the differential equation is of the form $yg_1(xy)dx+xg_2(xy)dy=0$	$I.F.=\frac{1}{Mx-Ny}$ *with* $Mx-Ny\neq 0$
Case 3: If $\frac{1}{N}\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right)=f(x)\ or\ C$	$I.F.=e^{\int f(x)dx}\ or\ e^{\int Cdx}$
Case 4 : If $\frac{1}{M}\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)=f(y)\ or\ C$	$I.F.=e^{\int f(y)dy}\ or\ e^{\int Cdy}$
Case 5 : If the differential equation is of the form $x^n y^n(mydx+nxdy)+x^{p'}y^{q'}(m'ydx+n'xdy)=0$	$I.F.=x^h y^k$ With $\frac{p+h+1}{m}=\frac{q+k+1}{n}$ *and* $\frac{p'+h+1}{m'}=\frac{q'+k+1}{n'}$
Newton's law of cooling	$T=T_o+C_1e^{-kt}$

Linear Algebra:

Inverse of a square matrix A

$$A^{-1}=\frac{(adj\ A)}{|A|}.$$

Rank of a Matrix A

The number of non-zero rows in the echelon form of A is equal to rank of A

Normal Form of a Matrix.

$$\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$$

Gauss-Elimination Method

The system is reduced to upper triangular system from which the unknowns are found by back substitutions.

Gauss-Jordan Method

The system is reduced to diagonal system from which the unknowns are found by back substitutions.

Eigenvalues

Roots of the characteristic equation $|A-\lambda I|=0$

Eigen Vectors

Non-zero solution $x=x_i$ of $|A-\lambda I|x=0$

Diagonal form

$$D=P^{-1}AP=\begin{bmatrix}\lambda_1 & 0 & 0\\ 0 & \lambda_2 & 0\\ 0 & 0 & \lambda_3\end{bmatrix}$$

Computation of power of a square matrix A

$$A^n=P\,D^nP^{-1}$$

Canonical Form

$$V=\lambda_1y_1^2+\lambda_2y_2^2+\lambda_3y_3^2+.......+\lambda_ny_n^2$$

Nature, Rank and Index of Quadratic forms:

- **positive-definite** if all the eigenvalues of A are positive.
- **positive-semi definite** if all the eigenvalues of A are non-negative and at least one of the eigenvalues is zero.
- **negative-definite** if all the eigenvalues of A are negative.
- **negative-semi definite** if all the eigenvalues of A are negative and at least one of the eigenvalues is zero.
- **indefinite** if the matrix A has both positive and negative eigenvalues.
- **Rank** the number of non-zero terms
- **Index** the number of positive terms
- **Signature** the number of positive terms minus the number of negative terms

Laplace Transforms:

Laplace Transform of Standard Functions

$L\{1\} = \frac{1}{s}$	$L\{t^n\} = \frac{n!}{s^{n+1}}$, $n = 1, 2, 3.......$ Where n is a positive integer
$L\{e^{at}\} = \frac{1}{s-a}$	$L\{e^{-at}\} = \frac{1}{s+a}$
$L\{\sin at\} = \frac{a}{s^2+a^2}$	$L\{\cos at\} = \frac{s}{s^2+a^2}$
$L\{\sinh at\} = \frac{a}{s^2-a^2}$	$L\{\cosh at\} = \frac{s}{s^2-a^2}$
$L\{u(t)\} = \frac{1}{s}$	$L\{u(t-a)\} = \frac{e^{-as}}{s}$

Properties of the Laplace transform:

$f(t)$	$L\{f(t)\} = F(s)$
Translation (first Shifting Theorem)	$L\{e^{at} f(t)\} = F(s-a)$
Multiplication by t	$L\{t^n \ f(t)\} = (-1)^n \frac{d^n}{ds^n}\{F(s)\}$
Time scale	$L\{f(at)\} = \frac{1}{a} F\left(\frac{s}{a}\right)$
Integration	$L\left\{\int_0^t f(t)\, d\tau\right\} = \frac{1}{s} L\{f(t)\}$
Division by t	$L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(s)\, ds.$

Transform of a Periodic Function: If $f(t)$ is periodic with a period T, then

$$L\{f(t)\} = \frac{1}{(1-e^{-sT})}\int_0^T e^{-st} f(t)\,dt \ .$$

Second Shifting Theorem:

If $F(s) = L\{f(t)\}$ **and** $a > 0$, **then** $L\{f(t-a)u(t-a)\} = e^{-as}L\{f(t)\}$.

Transforms of the derivatives:

$L\{f'(t)\} = sL\{f(t)\} - f(0)$

$L\{f''(t)\} = s^2L\{f(t)\} - s\,f(0) - f'(0)$

$L\{f^n(t)\} = s^nL\{f(t)\} - s^{n-1}f(0) - s^{n-2}f'(0) - s^{n-2}f''(0) - \cdots\cdots\cdots - f^{n-1}(0)$

Inverse Laplace Transform:

Transform $F(s) = L\{f(t)\}$	**Inverse Transform** $f(t) = L^{-1}\{F(s)\}$
$L\{t^n\} = \frac{n!}{s^{n+1}}$ Where n is a positive integer	$\frac{t^{n-1}}{(n-1)!} = L^{-1}\left\{\frac{1}{s^n}\right\}$ Where n is a positive integer
$L\{e^{at}\} = \frac{1}{s-a}$	$e^{at} = L^{-1}\left\{\frac{1}{s-a}\right\}$
$L\{\sinh at\} = \frac{a}{s^2-a^2}$	$\sinh at = L^{-1}\left\{\frac{a}{s^2-a^2}\right\}$
$L\{\cosh at\} = \frac{s}{s^2-a^2}$	$\cosh at = L^{-1}\left\{\frac{s}{s^2-a^2}\right\}$
$L\{\sin at\} = \frac{a}{s^2+a^2}$	$\sin at = L^{-1}\left\{\frac{a}{s^2+a^2}\right\}$
$L\{\cos at\} = \frac{s}{s^2+a^2}$	$\cos at = L^{-1}\left\{\frac{s}{s^2+a^2}\right\}$

Formulae on Shifting rule:

$L[e^{at}cosbt] = \frac{s-a}{(s-a)^2+b^2}$ $\qquad$ $L^{-1}[\frac{s-a}{(s-a)^2+b^2}] = e^{at}cosbt$

$L[e^{at}sinbt] = \frac{b}{(s-a)^2+b^2}$ $\qquad$ $L^{-1}[\frac{b}{(s-a)^2+b^2}] = e^{at}sinbt$

$L[e^{-at}cosbt] = \frac{s+a}{(s+a)^2+b^2}$ $\qquad$ $L^{-1}\left[\frac{s+a}{(s+a)^2+b^2}\right] = e^{-at}cosbt$

$L[e^{-at}sinbt] = \frac{b}{(s+a)^2+b^2}$ $\qquad$ $L^{-1}\left[\frac{b}{(s+a)^2+b^2}\right] = e^{-at}sinb$

$L[e^{at}sinhbt] = \frac{b}{(s-a)^2-b^2}$ $\qquad$ $L^{-1}\left[\frac{b}{(s-a)^2-b^2}\right] = e^{at}sinhbt$

$L[e^{at}coshbt] = \frac{s-a}{(s-a)^2-b^2}$ $\qquad$ $L^{-1}\left[\frac{s-a}{(s-a)^2-b^2}\right] = e^{at}coshbt$

$L[e^{-at}coshbt] = \frac{s+a}{(s+a)^2-b^2}$ $\qquad$ $L^{-1}\left[\frac{s+a}{(s+a)^2-b^2}\right] = e^{-at}coshbt$

$L[e^{-at}sinhbt] = \frac{b}{(s+a)^2-b^2}$ $\qquad$ $L^{-1}[\frac{b}{(s+a)^2-b^2}] = e^{-at}sinhbt$

Convolution Theorem:

Let $f(t)$ and $g(t)$ be piecewise continuous on $[0,\infty)$ and $F(s)=L\{f(t)\}$ & $G(s)=L\{g(t)\}$.

Then, $L^{-1}\{F(s)G(s)\}=\int_0^t f(u)g(t-u)du.$

Numerical Methods:

Regula Falsi formula: $x_{k+2} = x_k - \frac{x_{k+1}-x_k}{f(x_{k+1})-f(x_k)}$ for k = 0,1,2,………

Newton-Raphson formula:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$ for n = 1, 2, 3, ……..

Newton's Forward Interpolation formula:

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots\cdots\cdots$$

Where $p = \frac{x-x_0}{h}$

Newton's Backward Interpolation formula:

$$y_p = y_0 + p\nabla y_0 + \frac{p(p+1)}{2!}\nabla^2 y_0 + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_0 + \cdots\cdots\cdots$$

Where $p = \frac{x-x_n}{h}$

Newton's General Interpolation formula (Divided difference formula):

$$y = f(x) = y_0 + (x - x_0)[x_0\,, x_1] + (x - x_0)(x - x_1)[x_0\,, x_1\,, x_2] + (x - x_0)(x - x_1)(x - x_2)[x_0\,, x_1\,, x_2, x_3] + \cdots\cdots\cdots + (x - x_0)(x - x_1)\cdots\cdots(x - x_n)[x_0\,, x_1\,, \cdots\cdots, x_n]$$

Lagrange's Interpolation formula:

$$f(x) = \frac{(x - x_1)(x - x_2)(x - x_3)\cdots\cdots\cdots(x - x_n)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)\cdots\cdots\cdots(x_0 - x_n)} y_0 + \frac{(x - x_0)(x - x_2)(x - x_3)\cdots\cdots\cdots(x - x_n)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)\cdots\cdots\cdots(x_1 - x_n)} y_1 + \frac{(x - x_0)(x - x_1)(x - x_3)\cdots\cdots\cdots(x - x_n)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)\cdots\cdots\cdots(x_2 - x_n)} y_2 + \cdots\cdots\cdots\cdots + \frac{(x - x_0)(x - x_1)(x - x_2)\cdots\cdots\cdots(x - x_n)}{(x_n - x_0)(x_n - x_1)(x_n - x_2)\cdots\cdots\cdots(x_n - x_{n-1})} y_n$$

Numerical Integration:

Trapezoidal Rule:

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + \cdots\cdots + y_{n-1})]$$

Simpson's (1/3)rd rule:

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{3}[(y_0 + y_n) + 4(y_1 + y_3 + \cdots + y_{n-1}) + 2(y_2 + y_4 + \cdots + y_{n-2})]$$

Simpson's (3/8)th rule:

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{3h}{8}[(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5 + \cdots + y_{n-1}) + 2(y_3 + y_6 + \cdots + y_{n-3})]$$

Numerical methods for ODE's:

Taylor's series expansion about the point $x = x_0$.

$$y(x) = y(x_0) + \frac{(x-x_0)}{1!} y'(x_0) + \frac{(x-x_0)^2}{2!} y''(x_0) + \frac{(x-x_0)^3}{3!} y'''(x_0) + ..$$

Taylor's series expansion about the point $x = 0$

$$y(x) = y(0) + \frac{x}{1!} y'(0) + \frac{x^2}{2!} y''(0) + \frac{x^3}{3!} y'''(0) + \frac{x^4}{4!} y''''(0) + ...$$

Euler's Method:

$$y_{n+1}^{E} = y_n + hf(x_n, y_n), \qquad n = 0,1,2,3,\cdots\cdots$$

Modified Euler's Method:

$$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_{n+1}, y_{n+1}^{E}) \right], \qquad n = 0,1,2,3,\cdots\cdots$$

Runge–Kutta Method

Step1: Find $k_1 = hf(x_n, y_n)$; $k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2} \right)$

$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2} \right)$; $k_4 = hf(x_n + h, y_n + k_3)$

Step2: Find $y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

Milne's Method:

Step1: Find $f_1 = f(x_1, y_1)$, $f_2 = f(x_2, y_2)$ and $f_3 = f(x_3, y_3)$.

Step2: Predictor Formula $y_4^{(P)} = y_0 + \frac{4h}{3}[2f_1 - f_2 + 2f_3]$

Step3: Compute $f_4 = f(x_4, y_4^{(P)})$

Step4: Corrector Formula $y_4^{c} = y_2 + \frac{h}{3}\left(f_2 + 4f_1 + f_4^{P} \right)$

Adams – Bashforth Method:

Step1: Find $f_0 = f\left(x_0, y_0\right),\ f_1 = f(x_1, y_1),\ f_2 = f(x_2, y_2)$ and $f_3 = f(x_3, y_3)$.

Step2: **Predictor Formula** $y_4^P = y_3 + \frac{h}{24}\left(55f_3 - 59f_2 + 37f_1 - 9f_0\right)$

Step3: Compute $f_4 = f(x_4, y_4^{(P)})$

Step4: **: Corrector Formula** $y_4^c = y_3 + \frac{h}{24}\left(f_1 - 5f_2 + 19f_1 - 9f_4^P\right)$

Modular Arithmetics:

Set of Natural Numbers (N): {1, 2, 3, ...}

Set of Integers (I) = set of natural, negative naturals and zero: $\boldsymbol{N} \cup -\boldsymbol{N} \cup \{\mathbf{0}\}$

Greatest common divisor: $\boldsymbol{g\ or\ d = gcd(a, b)}$

Relative Primes: $\boldsymbol{g\ or\ d = gcd(a, b) = 1}$

Division Algorithm: For integers *a* and *b*, with *a* > 0, there exist integers *q* and *r*

such that $\boldsymbol{b = q \cdot a + r}$ **and** $\boldsymbol{0 \le r < a}$, **where *q*: *quotient, r: remainder***

6. Euclidean Algorithm

b	**a**	$\boldsymbol{b = q_1 a + r_1}$	$\boldsymbol{r_1 = b - a\, q_1}$
a	$\boldsymbol{r_1}$	$\boldsymbol{a = q_2 r_1 + r_2}$	$\boldsymbol{r_2 = a - q_2 r_1}$

***g = d· gcd (a, b) = ax + by* with *x* and *y* integers**

7. Modular and modular class: $\boldsymbol{m/(a-b)}$ **then** $\boldsymbol{a \equiv b(mod m),\ 0 \le |b| < m}$

$$\boldsymbol{b \equiv a(mod m),\ 0 \le |a| < m}$$

If $\boldsymbol{a \equiv b(mod m)}$ **then** $\boldsymbol{a - b = k \cdot m}$

8. Properties of modular arithmetic:

- $a \equiv a(mod m)$
- $a \equiv b(mod m)$ *and* $b \equiv c(mod m)$, then $a \equiv c(mod m)$
- $a \equiv c(mod m)$ *and* $b \equiv c(mod m)$, then $a \equiv b(mod m)$
- $a \equiv b(mod m)$ then for any $c, a + c \equiv (b + c)(mod m)$

- If $a \equiv b(mod m)$ then for any $c, a + c \equiv (b + c)(mod m)$
- If $a \equiv b(mod m)$ then for any c, $a \cdot c \equiv (b \cdot c)(mod m)$
- If $ac \equiv (bc)(mod m)$ and $d = \gcd(m, c)$, then $a \equiv b(mod\ m/d)$
- If $\boldsymbol{ac \equiv (bc)(mod m)}$ and m is a prime number, then $\boldsymbol{a \equiv b(mod\ m)}$
- If $a \equiv b(mod m)$ and $c \equiv d(mod m)$, then $a \pm c \equiv (b \pm d)(mod m)$
- If $a \equiv b(mod m)$ and $c \equiv d(mod m)$, then $a \cdot c \equiv (b \cdot d)(mod m)$
- If $\boldsymbol{ac \equiv b(mod m)\ and\ c \equiv d(mod m)}$, **then** $\boldsymbol{ab \equiv b(mod m)}$
- If $\boldsymbol{a \equiv b(mod m)},\ \boldsymbol{then\ a^n \equiv b^n(mod m)}$
- If $\boldsymbol{\gcd(m, n) = 1},\ \boldsymbol{a \equiv b(mod m)\ and\ a \equiv b(mod n)}$, then $\boldsymbol{a \equiv b(mod\ mn)}$
- If $\boldsymbol{ab \equiv 1(mod\ n), then\ a\ is\ inverse\ of\ b\ and\ b\ is\ the\ inverse\ of\ a}$

Linear Diophantine equation:

$$\boldsymbol{ax + by = c, with\ d = gcd(a, b)\ has\ solution\ if\ d/c}$$

(i) has d congruent solutions and if x_0 and y_0 are primitive solutions,

then $\boldsymbol{x = x_0 + \left(\frac{b}{d}\right)t},\ \boldsymbol{y = y_0 + (a/d)t}$, *for positive integer t*

(ii) if d = 1 then it has unique solution

Remainder theorem:

$$\boldsymbol{x \equiv a_i(mod\ m_i), i = 1, 2, 3,\ with\ (m_i, m_j) = 1,\ \ i \neq j}$$

Procedure to apply remainder theorem

If $\boldsymbol{m_i, i = 1, 2, 3}$ *are relatively prime*

Then the solution of $\boldsymbol{x_i \equiv b_i(mod\ m_i)},\ \ \boldsymbol{i = 1, 2,} 3$ *is* $\boldsymbol{x = \sum_1^3 b_i M_i y_i}$

where, $M_i = M / m_i$ *with* $\boldsymbol{M = \prod_1^3 m_i}$

$M_i \boldsymbol{y_i \equiv 1(mod\ mi}$) *where* $\boldsymbol{y_i\ is\ inverse\ of}$ M_i *under mod* m_i

11. Fermat's little theorem:

If p is any prime and p ∤a, then $\boldsymbol{a^{p-1}} \equiv 1\ (mod\ p)$ *or* $\boldsymbol{a^p} \equiv a\ (mod\ p)$

12. Euler's Φ function:

If n is any composite number with prime factorization $\boldsymbol{n} = \boldsymbol{p_1}^{m_1} X\ \boldsymbol{p_2}^{m_2} \boldsymbol{X} \ldots$ **;**
then number of primes up to n is

$$\boldsymbol{\Phi(n) = n\left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right)\cdots\cdots}$$

13. Wilson's theorem: **<u>Prime</u> *p* divides ($p - 1$)! + 1**

14. RSA cryptosystem:

If p and q are large prime numbers and a is any integer then,

plain text M is encrypted to c by, $\boldsymbol{c \equiv M^{e}(mod\ pq)}$

The public key will be = (pq, e)

d the decryption key the inverse of e, then $\boldsymbol{d \cdot e \equiv 1(mod(p-1)(q-1))}$

then $\boldsymbol{M \equiv c^{d}(modpq)}$.

Curvilinear Coordinates:

Curvilinear coordinates are often used to define the location or distribution of physical quantities which may be scalars, vectors or tensors.

If $\boldsymbol{\vec{R} = x(u,v,w)\hat{\imath} + y(u,v,w)\hat{\jmath} + z(u,v,w)\hat{k}}$ **,** then

(i) **Tangent vectors** to the u-curve, the v-curve and the w-curve are $\boldsymbol{\frac{\partial \vec{R}}{\partial u}, \frac{\partial \vec{R}}{\partial v}\ and\ \frac{\partial \vec{R}}{\partial w}}$ respectively

(ii) **Scale factors** $\boldsymbol{h_1 = \left|\frac{\partial \vec{R}}{\partial u}\right|, h_2 = \left|\frac{\partial \vec{R}}{\partial v}\right|\ and\ h_3 = \left|\frac{\partial \vec{R}}{\partial w}\right|}$

(iii) **Unit Tangent Vectors** to the u-curve, the v-curve and the w-curve are

$$T_u = \frac{\left(\frac{\partial \vec{R}}{\partial u}\right)}{h_1}, T_v = \frac{\left(\frac{\partial \vec{R}}{\partial v}\right)}{h_2}\ and\ T_w = \frac{\left(\frac{\partial \vec{R}}{\partial w}\right)}{h_3}$$, respectively.

Normal to the surfaces $\boldsymbol{u = u_0,\ v = v_0\ and\ w = w_0}$ are

$$\nabla u = \frac{T_u}{h_1}, \nabla v = \frac{T_v}{h_2}\ and\ \nabla w = \frac{T_w}{h_3}$$ respectively

Unit normal vectors to **the surfaces** $\boldsymbol{u = u_0, v = v_0\ and\ w = w_0}$ are

$$N_u = \frac{\nabla u}{|\nabla u|}, N_v = \frac{\nabla v}{|\nabla v|}\ \text{and}\ N_w = \frac{\nabla w}{|\nabla w|}$$, respectively

Polar Coordinates $(\boldsymbol{r, \theta})$ **:**

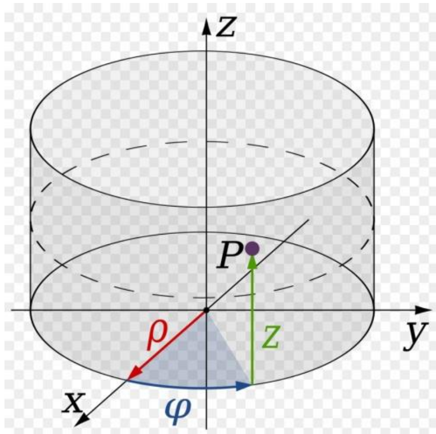
Coordinate transformations: $x = r\cos\theta,\ y = r\sin\theta$

Jacobian: $\frac{\partial(x,y)}{\partial(r,\theta)} = r$

$(Arc - length)^2 : (ds)^2 = (dr)^2 + r^2(d\theta)^2$

Cylindrical Coordinates (ρ, φ, z):

Coordinate transformations: $x = \rho \cos\varphi,\ y = \rho \sin\varphi, z = z$



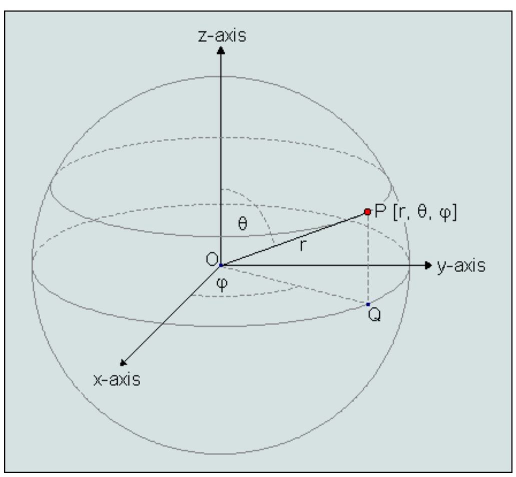
Jacobian: $\frac{\partial(x,y,z)}{\partial(\rho,\varphi,z)} = \rho$

$(Arc - length)^2$: $(ds)^2 = (d\rho)^2 + \rho^2(d\varphi)^2 + (dz)^2$

Volume element: $dV = \rho d\rho\, d\varphi\, dz$

Spherical Polar Coordinates (r, θ, φ) :

Coordinate transformations: $x = r\sin\theta\cos\varphi\ ,\ y = r\sin\theta\sin\varphi\ ,\ z = r\cos\theta$



Jacobian: $\frac{\partial(x,y,z)}{\partial(r,\theta,\varphi)} = r^2\sin\theta$

$(Arc - length)^2$: $(ds)^2 = (dr)^2 + r^2(d\theta)^2 + (r\sin\theta)^2(d\varphi)^2$

Volume element: $dV = r^2\sin\theta\, dr\, d\theta d\varphi$